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We investigate the spontaneous emission of radiation for a Bloch electron traversing a single energy miniband of a superlattice in an external homogeneous electric field subjected the influence of resonant microcavity and dephasing from an internal inhomogeneous electric field. The spontaneous emission for the cavity-enhanced Bloch electron probability amplitude becomes damped and frequency shifted due to the perturbing inhomogeneity when treated in a long-time, time-dependent perturbation theory relative to the Bloch-accelerated dynamics in the electrodynamic radiation field. The frequency shift is proportional to the diagonal matrix elements of the Hamiltonian for the perturbing inhomogeneity with respect to the instantaneous Bloch eigenstates, and the damping term is proportional to the off-diagonal matrix elements of the perturbing Hamiltonian with the instantaneous eigenstates summed to the final states as determined in a golden-rule like fashion. The resulting general theory is reduced for the specific cases of an abrupt and smoothly varying potentials but emphasis is given to the case of a "comb" of Slater-Koster impurities with randomly distributed interface roughness at all lattice sites. From the Slater-Koster case, the relaxation approximation is developed where the damping term is considered to be a constant and the frequency shift is ignored. Analysis of total power shows that dephasing degradation effects are more than compensated for by enhancements derived by microcavity confinement.

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Quantum analysis of a microcavity-tuned Bloch oscillator for tunable spontaneous emission and absorption of terahertz radiation

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Abstract

In this study, we investigate the spontaneous emission of radiation for a Bloch electron traversing a single energy miniband of a superlattice in an external homogeneous electric field subjected simultaneously the influence of resonant microcavity and dephasing effects from an internal inhomogeneous electric field. It is shown that the spontaneous emission for the cavity-enhanced Bloch electron probability amplitude becomes damped and frequency shifted due to the perturbing inhomogeneity when treated in a *long-time*, time-dependent perturbation theory relative to the Bloch-accelerated dynamics in the electrodynamic radiation field. The frequency shift is shown to be proportional to the diagonal matrix elements of the Hamiltonian for the perturbing inhomogeneity with respect to the instantaneous Bloch eigenstates, and the damping term is shown to be proportional to the off-diagonal transition matrix elements of the perturbing Hamiltonian with the instantaneous eigenstates summed to the appropriate final states as determined in a *golden-rule* like fashion. The resulting general theory is reduced for the specific cases of an abrupt and smoothly varying potentials but emphasis is given to the special case of a *comb* of Slater-Koster interface impurities with randomly distributed interface roughness at all lattice sites. From the Slater-Koster case, the *relaxation time approximation* is developed where the damping term is considered to be a constant and the frequency shift is ignored. Analysis of total power shows that dephasing degradation effects are more than compensated for by enhancements derived by microcavity confinement.

Contents

I. INTRODUCTION

A theory of microcavity enhanced spontaneous emission (SE) for a Bloch electron traversing a single energy band accelerating in an external constant electric field has been recently examined by the author and colleagues.[1] The theoretical analysis was fully quantum mechanical in that the quantized radiation field was described in terms of the dominant TE_{10} rectangular microcavity waveguide mode in the Coulomb gauge; also the instantaneous eigenstates of the Bloch Hamiltonian were utilized as the basis states in describing the Bloch electron dynamics to all orders in the constant electric field. Analysis of the probability amplitudes, over integral multiples of the Bloch period, resulted in *selection rules* for photon emission in both photon frequency and wave vector showing preferable transitions to the Wannier-Stark ladder levels; it was shown that the SE emission rate could be enhanced by tuning the emission frequency to align with the cavity mode spectral density peak, thus resulting in an output power of several microwatts in the terahertz spectral range for a GaAs-based superlattice imbedded in a microcavity.

In this study, the analysis of microcavity enhanced spontaneous emission (MESE) for a Bloch electron accelerating through a single miniband of a superlattice structure was extended to include the additional interaction of the electron with a perturbing inhomogeneous electric field arising from impurities or interface roughness inherent from the superlattice material process.[2] The intent in studying the effects of such perturbing inhomogeneities is to determine their role in limiting the MESE process due to scattering influences that dephase the coherency of the Bloch oscillation, and to establish a quantitative determination as to how the MESE selection rules are influenced by such perturbations. For that purpose, the Hamiltonian for the Bloch electron in the quantum electrodynamic field of interest is developed (Sec. II). The classical external electric field is described in the vector potential gauge, and the quantized electromagnetic radiation field is described by the dominant microcavity TE_{10} rectangular waveguide mode in the Coulomb gauge; the general inhomogeneous electric field is treated in the scalar potential gauge. In neglecting the higher-order quantum field-field interaction term, it is shown that the total Hamiltonian for this problem reduces to the sum of three contributions, the Hamiltonian for the Bloch electron in the classical external electric field interacting with the inhomogeneity, the Hamiltonian of the free quantized electromagnetic field, and the Hamiltonian for the first-order interaction between the cavity quantum field and the accelerated Bloch electron. Then, the instantaneous eigenstates of the Bloch Hamiltonian and the states of the free radiation field are utilized as basis states in describing the time development, and in calculating the SE transition rates of the accelerated Bloch electrons under the simultaneous perturbing action of the quantum cavity radiation field and the inhomogeneous potential energy (Sec. III). In treating the perturbing inhomogeneity in *long-time*, time-dependent perturbation theory relative to the Bloch-accelerated system in the electrodynamic radiation field, it is found that the SE amplitude for the cavity-enhanced Bloch electron radiation becomes damped and frequency shifted in the off diagonal and diagonal matrix elements of the inhomogeneous potential energy with respect to the instantaneous Bloch eigenstates. In Sec. IV, the general theoretical analysis is applied to the specific cases where the inhomogeneous potential is a slowly varying or, in contrast, an abruptly varying function of the coordinates as well as a *comb* of Slater-Koster impurities of varying strength positioned at all the lattice cites. Here it is found that the frequency shift is a constant, and the damping constant is slowly varying with the Brillouin zone vector. Therefore, for purposes of showing trends with regard to simple dephasing effects, the dephasing analysis is carried out in Sec. V for the case where the damping term is a constant and the frequency shift is ignored; also a numerical estimate of SE power is provided. In Sec. VI, a summary of the most important results is given; most importantly, our analysis of the total power shows that dephasing degradation effects are more than compensated for by the enhancements derived by microcavity-based confinement engineering. Section VII contains two Appendixes, Appendix A and Appendix B; in Appendix A, we provide the details of the time-dependent perturbation theory analysis used to calculate the probability amplitude, and Appendix B provides the details of calculation of matrix elements of perturbation Hamiltonian.

II. QUANTUM APPROACH

The dynamical properties are considered for the situation in which the electron is confined to a single miniband "n_0" of a superlattice with energy $\varepsilon_{n_0}(\mathbf{K})$ while the effects of interband coupling are ignored.[3] Therefore, the quantum dynamics is described by the time-dependent Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}|\Psi_{n_0}(t)\rangle = H|\Psi_{n_0}(t)\rangle \, , \tag{1}$$

where the exact Hamiltoian $H = [\mathbf{p} - (e/c)\mathbf{A}]^2/2m_0 + V_c(\mathbf{r}) + H_r + V(\mathbf{r}, t)$ can be reduced to a sum of the following separate Hamiltonians $H = H_0 + V(\mathbf{r}, t) + H_r + H_I$. Here, the first two terms represent the Hamiltonian, $H_0(t) = [\mathbf{p} + \mathbf{p}_c(t)]^2/2m_0 + V_c(\mathbf{r})$, for single electron in a periodic crystal potential, $V_c(\mathbf{r})$, interacting with a homogeneous electric field $\mathbf{E}$, and $V(\mathbf{r}, t)$ is the potential energy for the inhomogeneous electric field due to the impurities or interface roughness; also H_r is the Hamiltonian for the cavity mode electromagnetic radiation field (m_0 is the free-electron mass, c is the velocity of light in vacuum). The total vector potential in the exact Hamiltonian consists of $\mathbf{A} = \mathbf{A}_c + \mathbf{A}_r$, where $\mathbf{A}_c = -(c/e)\mathbf{p}_c$ describes the external electric field with $\mathbf{p}_c(t) = e\int_{t_0}^{t} \mathbf{E}(t')dt' = eEt$ for the time-independent homogeneous electric field turned on at initial time $t_0 = 0$; $\mathbf{A}_r$ describes the TE_{10} cavity mode of the quantized radiation field $\mathbf{E}_r$ with the frequency ω_q given by

$$A_{r,y} = \sum_{q_z} \sqrt{\frac{4\pi\hbar c^2}{\omega_q \varepsilon V}}\, sin(q_x x) \left(\hat{a}_q e^{iq_z z} + \hat{a}_q^\dagger e^{-iq_z z}\right) , \tag{2}$$

where $\hat{a}_q^\dagger$ and $\hat{a}_q$ are the photon boson creation and annihilation operators, ε is the dielectric constant of the medium filling the waveguide of the length L_z and cross section $L_x \times L_y$, $V = L_x L_y L_z$, and $q_x = \pi/L_x$. For the chosen system geometry, the corresponding electric field $\mathbf{E}_r$ is polarized in the direction of the dc field, which is assumed to be along the y axis (also the superlattice growth direction). The normalization constant in Eq. (2) is chosen in such a way that the Hamiltonian for the quantized radiation field has the form $H_r = \sum_q \hbar\omega_q \hat{a}_q^\dagger \hat{a}_q$, where $\omega_q = \omega_c[1 + (q_z/q_x)^2]^{1/2}$ is the mode dispersion relation, and $\omega_c = q_x c/\sqrt{\varepsilon}$ is the angular cutoff frequency. The guided mode wavelength is written as $\lambda = \lambda_c/[(\omega_q/\omega_c)^2 - 1]^{1/2}$, where $\lambda_c = 2L_x$ is the cutoff wavelength.[4] The Hamiltonian $H_I(t) = -(e/m_0 c)\mathbf{A}_r \cdot [\mathbf{p} + \mathbf{p}_c(t)]$, for the first-order interaction between the quantum field and the Bloch electron, couples both subsystems H_0 and H_r, and causes transitions between the accelerated Bloch electron states through photon absorption and emission. Then, starting with the reduced Hamiltonian $H = H_0 + V(\mathbf{r}, t) + H_r + H_I$, use is made of first-order, *early-time* time-dependent perturbation theory[5] to calculate SE transitions probabilities between states of $H_0 + H_r$ while regarding $H_I(t) \sim \mathbf{A}_r \cdot [\mathbf{p} + \mathbf{p}_c(t)]$ as a perturbation, and at the same time, use is made of a *long-time* perturbation theory analysis[6] to calculate the relaxation influence of $V(\mathbf{r}, t)$ on the SE transition probabilities. This time scaling is justified since SE rates are orders of magnitude faster than relaxation times due to $V(\mathbf{r}, t)$.

III. QUANTUM DYNAMICS BASED ON INSTANTANEOUS EIGENSTATES

The solution for $|\Psi_{n_0}(t)\rangle$ in Eq. (1) can be represented in terms of the eigenstates of basis states $|\psi_{n_0\mathbf{k}(t)}, \{n_{\mathbf{q},j}\}\rangle = |\psi_{n_0\mathbf{k}(t)}\rangle\, |\{n_{\mathbf{q},j}\}\rangle$ of the unperturbed Hamiltonian $H_0 + H_r$ as

$$|\Psi_{n_0}(t)\rangle = \sum_{\mathbf{k}} \sum_{\{n_{\mathbf{q},j}\}} A_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t)|\psi_{n_0\mathbf{k}(t)}, \{n_{\mathbf{q},j}\}\rangle$$
$$\times exp\left\{-\frac{i}{\hbar}\int_{t_0}^{t}[\varepsilon_{n_0}(\mathbf{k}(t')) + \sum_{\mathbf{q},j}\hbar\omega_q n_{\mathbf{q},j}]dt'\right\}\,, \tag{3}$$

where the summation over $\mathbf{k}$ is carried out over the entire Brillouin zone, and $\{n_{\mathbf{q},j}\}$ is specified over all possible combinations of photon occupation number $n_{\mathbf{q},j}$ with photon wave vectors $\mathbf{q}$ and polarization $j = 1, 2$. The instantaneous eigenstates of H_0 are given[3] by $\psi_{n_0\mathbf{k}(t)}(\mathbf{r},t) = \Omega^{-1/2}e^{i\mathbf{K}\cdot\mathbf{r}}u_{n_0\mathbf{k}(t)}(\mathbf{r},t)$, where $u_{n_0\mathbf{k}(t)}(\mathbf{r},t)$ is the periodic part of the Bloch function, $\mathbf{k}(t) = \mathbf{K} + \mathbf{p}_c(t)/\hbar$, and the values of the electron wave vector $\mathbf{K}$ are determined by the periodic boundary conditions of the periodic crystal of volume Ω.

For the case of one-photon SE, which assumes that initially no photons are present in the field, the probability amplitude in the wave function of Eq. (3) satisfies the initial condition $A_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t_0) = \{\delta_{n_{\mathbf{q},j},0}\}\,\delta_{\mathbf{K},\mathbf{K}_0}$ at time $t = t_0$ when the electric field is turned on. Here, $\mathbf{K}_0$ and $n^0_{\mathbf{q},j} = 0$ are the initial values of $\mathbf{K}$ and $n_{\mathbf{q},j}$. Thus, the instantaneous eigenstates of H_0 satisfy the equation[3]

$$\left\{\frac{1}{2m_0}\left[\mathbf{p} + \mathbf{p}_c(t)\right]^2 + V_c(\mathbf{r})\right\}\psi_{n_0\mathbf{k}(t)} = \varepsilon_{n_0}(\mathbf{k}(t))\psi_{n_0\mathbf{k}(t)}\,, \tag{4}$$

and the basis states for H_r are given by the well-known free photon field equation

$$H_r|\{n_{\mathbf{q},j}\}\rangle = \sum_{\mathbf{q},j}\hbar\omega_q n_{\mathbf{q},j}|\{n_{\mathbf{q},j}\}\rangle\,, \tag{5}$$

where $|\{n_{\mathbf{q},j}\}\rangle$ is a simple product of all possible combinations of photon number states, $n_{\mathbf{q},j}$, with a given wave vector $\mathbf{q}$ and polarization $\hat{\varepsilon}_{\mathbf{q},j}$.

The appropriate time-dependent equations of motion for the $A_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t)$ coefficients expressed in Eq. (3) can be found in Appendix A; these equations relate the time rate of change of $A_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t)$ to the basis-dependent matrix elements of H_I and $V(\mathbf{r},t)$ through a self-consistent set of equations. In applying *early-time*, first-order perturbation theory to the "H_I" coefficients of the set of equations, and applying a *long-time*, Wigner-Weiskopf-like approximation to the "V" coefficients of the set of equations, we obtain (see Appendix A) a closed form inhomogeneous equation for the one-photon SE amplitude [here $\{n_{\mathbf{q},j}\} \rightarrow n = 1$] at $\mathbf{k}_0(t) = \mathbf{K}_0 + \mathbf{p}_c(t)/\hbar$ as

$$\dot{A}_n(\mathbf{k}_0, t) - \frac{1}{i\hbar} f_{\mathbf{k}_0\mathbf{k}_0}(t) A_n(\mathbf{k}_0, t) + \frac{1}{\hbar^2}\int_0^t dt' \sum_{\mathbf{k}' \neq \mathbf{k}_0} f_{\mathbf{k}_0\mathbf{k}'}(t) f^*_{\mathbf{k}_0\mathbf{k}'}(t') A_n(\mathbf{k}_0, t')$$
$$= \dot{A}^0_q(\mathbf{k}_0, t) + \frac{1}{i\hbar} \sum_{\mathbf{k}' \neq \mathbf{k}_0} f_{\mathbf{k}_0\mathbf{k}'}(t) A^0_q(\mathbf{k}', t) \, . \qquad (6)$$

Here,

$$f_{\mathbf{k}_0\mathbf{k}'}(t) = V_{\mathbf{k}_0\mathbf{k}'} \, exp\left\{ \frac{i}{\hbar}\int_0^t \{\varepsilon_{n_0}[\mathbf{k}_0(t')] - \varepsilon_{n_0}[\mathbf{k}'(t')]\} dt' \right\}, \qquad (7)$$

where $V_{\mathbf{k}_0\mathbf{k}'} = (\psi_{n_0\mathbf{k}_0(t)}, V\psi_{n_0\mathbf{k}'(t)})$ are the appropriate matrix elements of the perturbing inhomogeneity $V(\mathbf{r}, t)$, and

$$A^0_q(\mathbf{k}_0, t) = D(q_x/q)^{1/2} \int_0^t dt' \, v_y[\mathbf{k}_0(t') - \mathbf{q}_s]$$
$$\times exp\left\{ -\frac{i}{\hbar}\int_0^{t'} \{\varepsilon_{n_0}[\mathbf{k}_0(t_1)] - \varepsilon_{n_0}[\mathbf{k}_0(t_1) - \mathbf{q}_s] - \hbar\omega_q\} dt_1 \right\}, \qquad (8)$$

the probability amplitude for the microcavity-based SE alone.[1] Also, $D = -i\sqrt{\pi c\alpha/\omega_c \varepsilon V}$, $\alpha = e^2/\hbar c$ is the fine structure constant, $\mathbf{q}_s = \{\pm q_x, 0, q_z\}$ with " + " for $s = 1$ and " − " for $s = 2$, $v_y(\mathbf{k}(t)) = (1/\hbar)\nabla_{\mathbf{K_y}}\varepsilon_{n_0}(\mathbf{K})|_{\mathbf{k}(t)}$, the y component of Bloch velocity in the band, and $q = (q_x^2 + q_z^2)^{1/2}$.

IV. ANALYSIS OF DEPHASING FOR PARTICULAR MODELS OF INHOMOGENEITY

The solution for $A_n(\mathbf{k}_0, t)$ in Eq. (6) depends explicitly on the potential energy $V(\mathbf{r}, t)$, through $f_{\mathbf{k}_0\mathbf{k}'}(t)$ as noted in Eq. (7). In general, the time dependence of $f_{\mathbf{k}_0\mathbf{k}'}(t)$, expressed through the time dependent matrix elements of $V(\mathbf{r}, t)$ and through the time dependence of accelerated instantaneous energy eigenstates in the phase of $f_{\mathbf{k}_0\mathbf{k}'}(t)$, makes finding the solutions for $A_n(\mathbf{k}_0, t)$ in Eq. (6) formidable. However, for the type of potential energy function of interest in this problem, that is, one where $V(\mathbf{r}, t) \equiv V(\mathbf{r})$, a function of position alone, and where $V(\mathbf{r})$ is a *comb* of abruptly changing, Slater-Koster like, interface impurities with randomly distributed interface roughness, it can be shown[7] that $f_{\mathbf{kk}'}(t)$ is expressed as

$$f_{\mathbf{kk}'}(t) = V_{\mathbf{KK}'} \, e^{-\frac{i}{\hbar}[\eta(\mathbf{K}) - \eta(\mathbf{K}')]} \, e^{\frac{i}{\hbar}[\bar{\varepsilon}_{n_0}(\mathbf{K}_\perp) - \bar{\varepsilon}_{n_0}(\mathbf{K'}_\perp)]t}. \qquad (9)$$

It is important to note that $V_{\mathbf{KK}'}$, $\eta(\mathbf{K})$, and $\bar{\varepsilon}_{n_0}(\mathbf{K}_\perp)$ are all *time independent.* $V_{\mathbf{KK}'}$ are the matrix elements of the interface inhomogeneities; $\eta(\mathbf{K})$ and $\bar{\varepsilon}_{n_0}(\mathbf{K}_\perp)$ are band structure dependent *only*, and are given by

$$\eta(\mathbf{K}) = \sum_{l_y=1}^{\infty} \frac{2\,\epsilon_{n_0}(al_y, \mathbf{K}_\perp)}{\omega_B l_y} sin(K_y a l_y), \tag{10}$$

with

$$\epsilon_{n_0}(al_y, \mathbf{K}_\perp) = \frac{1}{N_y} \sum_{K_y} \varepsilon_{n_0}(K_y, \mathbf{K}_\perp) e^{-iK_y a l_y}\,, \tag{11}$$

and

$$\bar{\varepsilon}_{n_0}(\mathbf{K}_\perp) = \frac{1}{N_y} \sum_{K_y} \varepsilon_{n_0}(K_y, \mathbf{K}_\perp)\,, \tag{12}$$

the average value of $\varepsilon_{n_0}(\mathbf{K})$ along the K_y direction in the Brillouin zone, independent on K_y; N_y is the number of lattice sites along the y axis. For nearest-neighbor tight-binding considerations, only the term with $l_y = 1$ need be retained in Eqs. (10) and (11). Then, in utilizing $f_{\mathbf{kk}'}(t)$ from Eq. (9) in Eq. (6), and anticipating the ensemble average over the randomly distributed interface roughness coordinates, which renders the ensemble average over the $f_{\mathbf{kk}'}(t)$ term on the right-hand side of Eq. (6) equal to zero, then the ensemble averaged equation for $A_n(\mathbf{k}_0, t)$ reduces to

$$\dot{A}_n(\mathbf{k}_0, t) - \frac{1}{i\hbar} V_{\mathbf{k}_0\mathbf{k}_0} A_n(\mathbf{k}_0, t) + \frac{1}{\hbar^2} \sum_{\mathbf{k}' \neq \mathbf{k}_0} |V_{\mathbf{k}_0\mathbf{k}'}|^2\, e^{-\frac{i}{\hbar}[\eta(\mathbf{K}_0) - \eta(\mathbf{K}')]}$$
$$\times \int_0^t dt'\, e^{\frac{i}{\hbar}[\bar{\varepsilon}_{n_0}(\mathbf{K}_{0\perp}) - \bar{\varepsilon}_{n_0}(\mathbf{K'}_\perp)](t-t')} A_n(\mathbf{k}_0, t') = \dot{A}_q^0(\mathbf{k}_0, t)\,. \tag{13}$$

This is an inhomogeneous differential equation in time for $A_n(\mathbf{k}_0, t)$, where the inhomogeneity on the right-hand side is the time derivative of the SE for the cavity-based result alone [see Eq. (8)]. This equation can be readily solved by the Laplace transform method in the long-time limit to obtain

$$A_n(\mathbf{k}_0, t) = \int_0^t dt'\, \dot{A}_q^0(\mathbf{k}_0, t')\, e^{-i\Delta\omega(\mathbf{k}_0)t'}\, e^{-\frac{\Gamma(\mathbf{k}_0)}{2}t'}\,, \tag{14}$$

where

$$\Delta\omega(\mathbf{k}_0) = \frac{1}{\hbar} V_{\mathbf{k}_0\mathbf{k}_0} + \frac{1}{\hbar} \sum_{\mathbf{k}' \neq \mathbf{k}_0} \frac{|V_{\mathbf{k}_0\mathbf{k}'}|^2}{\bar{\varepsilon}_{n_0}(\mathbf{K}_{0\perp}) - \bar{\varepsilon}_{n_0}(\mathbf{K'}_\perp)}\, e^{\frac{2}{\hbar} Im[\eta(\mathbf{K}_0) - \eta(\mathbf{K}')]}\,, \tag{15}$$

and

$$\Gamma(\mathbf{k}_0) = \frac{2\pi}{\hbar} \sum_{\mathbf{k}' \neq \mathbf{k}_0} |V_{\mathbf{k}_0\mathbf{k}'}|^2\, e^{\frac{2}{\hbar} Im[\eta(\mathbf{K}_0) - \eta(\mathbf{K}')]}\, \delta[\bar{\varepsilon}_{n_0}(\mathbf{K}_{0\perp}) - \bar{\varepsilon}_{n_0}(\mathbf{K'}_\perp)]\,. \tag{16}$$

This result for $A_n(\mathbf{k}_0, t)$ in Eq. (14) shows that the SE for the cavity-enhanced Bloch electron probability amplitude becomes damped by $\Gamma(\mathbf{k})$ in Eq. (16) and frequency shifted by $\Delta\omega(\mathbf{k})$ given in Eq. (15) due to the perturbing influence of the assumed interface inhomogeneity and the spatially localizing effect of the constant electric field from $\eta(\mathbf{K})$ defined in Eq. (10). It is interesting to note that the result obtained in Eq. (14) is strikingly reminiscent of the result obtained for the solution of the kinetic Boltzmann equation[8] for the carrier distribution function in the relaxation time approximation. Therefore, in an approach similar to analyzing the complications of the Boltzmann theory, we first consider the model, noted as the relaxation time approximation, where $\Delta\omega \equiv 0$ and $\Gamma(\mathbf{k})$ is a constant, independent of $\mathbf{k}$, to analyze, in the simplest heuristic approximation, the degrading effects of dephasing.

V. RELAXATION TIME APPROXIMATION

In this section, for purposes of showing heuristic trends with regard to simple dephasing effects, the dephasing analysis is carried out for the case where the damping term is a constant and the frequency shift is ignored. Then, it follows from Eqs. (14)-(16) that the one-photon SE probability amplitude [i.e., $\{n_{\mathbf{q},j}\} \to n = 1$] can be presented as

$$A_q(\mathbf{k}_0, t) = D(q_x/q)^{1/2} \int_{t_0}^{t} dt'\, v_y(\mathbf{k}_0 - \mathbf{q}_s) e^{-(t'-t_0)/\tau}$$
$$\times exp\left\{ -\frac{i}{\hbar} \int_{t_0}^{t'} [\varepsilon_{n_0}(\mathbf{k}_0) - \varepsilon_{n_0}(\mathbf{k}_0 - \mathbf{q}_s) - \hbar\omega_q] dt_1 \right\} , \qquad (17)$$

where τ has the meaning of the characteristic mean dephasing time. Equation (17) can be formally rewritten as

$$A_q(\mathbf{k}_0, t) = D(q_x/q)^{1/2} \int_{t_0}^{t} dt'\, v_y(\mathbf{k}_0 - \mathbf{q}_s)$$
$$\times exp\left\{ -\frac{i}{\hbar} \int_{t_0}^{t'} [\varepsilon_{n_0}(\mathbf{k}_0) - \varepsilon_{n_0}(\mathbf{k}_0 - \mathbf{q}_s) - \hbar\tilde{\omega}_q] dt_1 \right\} , \qquad (18)$$

with a renormalized complex photon "energy" $\hbar\tilde{\omega}_q = \hbar\omega_q + i\hbar\omega_\tau$, where $\omega_\tau = 1/\tau$. Thus the emission process results in the SE probability

$$P_e^s(t) = \sum_q \sum_{s=1,2} |A_q(\mathbf{k}_0, t)|^2 . \qquad (19)$$

A. Selection rules

In evaluating $A_q(\mathbf{k}_0, t)$, we take into account that the external dc field, $\mathbf{E}$, is along the y axis; then, it follows that $k_{0y}(t) = K_{0y} + eE(t - t_0)/\hbar$. In taking advantage of the periodic properties of the terms in Eq. (17), $A_q(\mathbf{k}_0, t)$ is evaluated in *clocked* integral multiples of the Bloch period, so that $t = N\tau_B$, where $\tau_B = 2\pi/\omega_B$, the time to traverse one period of the Brillouin zone. The integral in Eq. (17) over time can be replaced by an integral over k_{0y} through the substitution $dt = (\hbar/eE)dk_{0y}$. Then the probability amplitude, at integral multiples of the Bloch period, can be expressed through that over the single Bloch period, τ_B.[3,9] Thus we obtain

$$A_{\mathbf{q}}(\mathbf{k}_0, N\tau_B) = \left[\frac{1 - exp(-iN\tilde{\beta}_q)}{1 - exp(-i\tilde{\beta}_q)}\right] A_{\mathbf{q}}(\mathbf{k}_0, \tau_B)\,, \tag{20}$$

where the complex parameter $\tilde{\beta}_q$ is given by

$$\tilde{\beta}_q = 2\pi\frac{\tilde{\omega}_q}{\omega_B} + \frac{1}{eE}\int_{K_{0y}}^{K_{0y}+G_y} dk_{0y}\left[\varepsilon_{n_0}(\mathbf{k}_0) - \varepsilon_{n_0}(\mathbf{k}_0 - \mathbf{q}_s)\right]\,, \tag{21}$$

and $G_y = 2\pi/a$, the y component of the SL reciprocal-lattice vector. From Eq. (20) the corresponding squared probability amplitudes are expressed as

$$|A_q(\mathbf{k}_0, N\tau_B)|^2 = \eta(\omega_q; N, \tau)\,|A_q(\mathbf{k}_0, \tau_B)|^2\,, \tag{22}$$

where the "transfer" function $\eta(\omega_q; N, \tau)$ is given by

$$\eta(\omega_q; N, \tau) = \frac{sh^2(\pi N/\tau\omega_B) + sin^2(\pi N\omega_q/\omega_B)}{sh^2(\pi/\tau\omega_B) + sin^2(\pi\omega_q/\omega_B)}e^{-2\pi(N-1)/\tau\omega_B}\,. \tag{23}$$

From Eqs. (22) and (23), it is seen that for the limiting case of $\pi N/\tau\omega_B \ll 1$, i.e., when the electron scattering is not essential even for many Bloch oscillations (N) in the time scale of the scattering time τ, the quantity $|A_q(\mathbf{k}_0, N\tau_B)|^2$ will reach its maximum growth value when $\beta_q = 2\pi(m + \delta)$, where $\beta_q = Re(\tilde{\beta}_q)$, m is an integer and $\delta \to 0$; for this limit, the function $\eta(\omega_q; N, \tau)$ is reduced to the function $\eta(\omega_q, N)$ previously obtained in the absence of scattering:[9] $\eta(\omega_q, N) = \sin^2(N\beta_q/2)/\sin^2(\beta_q/2) \to N^2$, i.e., it becomes sharply peaked at the resonances with increasing N. It is clear that this condition for maximum growth establishes the *selection rule*[3,9] for both the photon emission frequency, ω_q, and the key wave vector component, q_z. Indeed, from the condition $\beta_q = 2\pi m$, it follows from Eq. (21) in the radiative long-wavelength limit ($qa \ll 1$) that one generally[10] has

$$\omega_q = m\omega_B, \quad q_z = q_{zm} \equiv q_x \left[\left(m\frac{\omega_B}{\omega_c} \right)^2 - 1 \right]^{1/2} ; \tag{24}$$

this shows the *"Stark ladder"* resonance frequency condition, along with the sustaining wave vector cavity resonance conditions. Thus, the modes that radiate with the highest probability correspond to the fundamental Bloch frequency and its harmonics. These quantization conditions are obtained without requiring any assumptions concerning the existence of Wannier-Stark energy states. In Figs. 1 and 2, we show the modification of the behavior of $\eta(\omega_q; N, \tau)$ with increasing the intensity of scattering (decreasing τ) calculated for reasonable scattering ($N\tau_B/\tau = 1$) and strong scattering ($N\tau_B/\tau = 4$), respectively, with $N = 10$. It is seen that the peak positions are negligibly affected, although the peak spectral width becomes broadening and the peak value is reduced with decreasing ratio τ/τ_B, as shown in Figs. 3 and 4. From this result, it is noted that the maximum growth condition for the probability amplitudes in Eqs. (22) and (23) still establishes the selection rules for spontaneous emission given in Eq. (24) despite the increasing intensity of scattering ($\tau\nu_B \gtrsim 1$).

It is worth to emphasize that in what follows the equation (22) should be considered simultaneously with the equation (24), i.e., both established resonance conditions for the emission photon frequency and wave vector should be taken into account explicitly in evaluating the spontaneous emission probability spectra given by $|A_q(\mathbf{k}_0, \tau_B)|^2$.

B. General expression for the total SE probability

The total spontaneous emission probability is evaluated at time $t = N\tau_B$, $P_e^s = P_e^s(N\tau_B)$, by substituting $|A_q(\mathbf{k}_0, N\tau_B)|^2$ from Eq. (22) into Eq. (19). The sum over q in Eq. (19) has been replaced by an integral over q, taking into account the TE_{10} mode density of states and polarization such that $\sum_q(\cdots) \to (L_z/\pi)\int dq(\cdots)/[1-(q_x/q)^2]^{1/2}$. Thus we can write

$$P_e^s = \frac{L_z}{\pi} \sum_{s=1}^{2} \int dq\, \eta(\omega_q; N, \tau) \frac{|A_q(\mathbf{k}_0, \tau_B)|^2}{[1-(q_x/q)^2]^{1/2}} \,. \tag{25}$$

The integral can be evaluated by using the property of the integrand which contains a sharply peaked, symmetric function of q, $\eta(\omega_q; N, \tau)$, at $q = q_m \equiv (q_x^2 + q_{zm}^2)^{1/2}$ [see Eq. (24)]. We note that with decreasing the scattering time τ, the maximum values of this function are suppressed relatively its peak value, N^2, corresponding to an infinite τ (Figs. 3 and 4). However, its sharp and oscillatory behavior still preserved for the scattering time not much less than the inverse Bloch frequency ν_B, i.e., for $\tau\nu_B \gtrsim 1$ (Figs. 1 and 2). In what follows we assume such values of τ and ν_B that satisfy this inequality. Thus, at every node defined by the resonance conditions, the slowly varying function of q in the integrand can be replaced by its value evaluated at $q = q_m$, and then removed from the integral over q; after that, we obtain

$$P_e^s = I_N(\tau\nu_B)\frac{L_z\omega_B}{L_x\omega_c}\sum_{l=1}^{l_{max}}\sum_{s=1}^{2}\frac{|A_{q_l}(\mathbf{k}_0,\tau_B)|^2}{[1-(q_x/q_l)^2]^{1/2}}\,. \tag{26}$$

Here l_{max} follows from $q_{max} = l_{max}(\omega_B/\omega_c)q_x$, and determines the upper limit in the sum over higher Bloch oscillation harmonics. The integral $I_N(\tau\nu_B)$ is determined by

$$I_N(\tau\nu_B) = e^{-(N-1)/\tau\nu_B}\frac{2}{\pi}\int_0^{\pi/2} dx\frac{sh^2(N/2\tau\nu_B)+sin^2(Nx)}{sh^2(1/2\tau\nu_B)+sin^2(x)}\,. \tag{27}$$

The calculation of P_e^s in Eq. (26) requires the use of $A_q(\mathbf{k}_0,\tau_B)$ in Eq. (17), evaluated at the maximum growth conditions of Eq. (24), that is when $\hbar\omega_q = m\hbar\omega_B$ and $q = q_m$. In addition, the dependence upon $\mathbf{q}$ in Eq. (17) is made explicit by invoking the assumption of photon long-wavelength limit, which is valid for all periodic potentials of interest, even superlattices, where $q \ll \pi/a$. Thus, we find that

$$A_{q_l}(\mathbf{k}_0,\tau_B) = -\frac{2\pi}{\omega_B}\left(\frac{q_x}{q_l}\right)^{1/2} DI_l\, exp(iK_{0y}a\tilde{\omega}_{q_l}/\omega_B)\,, \tag{28}$$

where

$$I_l = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\vartheta_k[v_y(\vartheta_k)exp(\vartheta_k/2\pi\tau\nu_B)]\, exp(-il\vartheta_k) \tag{29}$$

is the l-th Fourier component of the function $v_y(\vartheta_k)\, exp(\vartheta_k/2\pi\tau\nu_B)$, $\vartheta_k = k_{0y}a$, and $\tilde{\omega}_{q_l} = l\omega_B + i/\tau$. Since the electron velocity component $v_y(\vartheta_k)$ depends on details of the superlattice miniband structure, such dependence comes explicitly into the SE probability amplitude through the integral I_l given by Eq. (29); thus

$$|A_{q_l}(\mathbf{k}_0,\tau_B)|^2 = \frac{q_x}{q_l(\omega_B/2\pi)^2}|DI_l|^2\, exp(-K_{0y}a/\pi\tau\nu_B)\,. \tag{30}$$

C. Analysis of the total SE probability for specific energy miniband

The analysis for spontaneous emission and radiation characteristics is now developed by considering a quite general form of the electron energy miniband dispersion relation expressed as

$$\varepsilon_{n_0}(\mathbf{K}) = \varepsilon_{n_0}(0) + \sum_{l'=1}^{\infty} \Delta_{l'} \sin^2 \frac{l' a K_y}{2} + \varepsilon_{\perp}(\mathbf{K}_{\perp}) , \tag{31}$$

where $\varepsilon_{n_0}(0)$ is the miniband edge, $\Delta_{l'}$ is the width of the l'-th miniband harmonic of the superlattice, and $\varepsilon_{\perp}(\mathbf{K}_{\perp})$ is the contribution from the perpendicular components of the band. Such form of the energy band dispersion in the superlattice growth direction generally includes long range coupling over the neighboring QWs with a relative strength measured by the specific value of the ratio $\Delta_{l'+1}/\Delta_{l'} < 1$, which is strongly dependent upon the extent of wave function overlap. In particular, the well-known case of nearest-neighbor tight-binding (NNTB) approximation corresponds with purely harmonic energy dispersion where only the single term (l'=1) is considered significant, so that next nearest neighbor and longer range QW wave function overlaps are assumed to be negligibly small. The electron group velocity in the general miniband of Eq. (31), for the given K_y in the y direction, is then given by $v_y(K_y) = (1/\hbar)(\partial\varepsilon_{n_0}(K_y)/\partial K_y) = \sum_{l'=1}^{\infty} v_{l'} \sin(l' a K_z)$, where $v_{l'} = l' a \Delta_{l'}/2\hbar$, the maximum velocity associated with the l'-th miniband of band width, $\Delta_{l'}$. Substituting the expression for $v_y(K_y)$ in Eq. (29), one can find

$$I_l = \frac{1}{2\pi i}\, v_1 S_l \, sh(1/2\tau\nu_B) , \tag{32}$$

where

$$S_l = \sum_{l'=1}^{\infty} \frac{v_{l'}}{v_1} \left\{ \frac{cos[\pi(l'-l)]}{i(l'-l) + 1/2\pi\tau\nu_B} + \frac{cos[\pi(l'+l)]}{i(l'+l) - 1/2\pi\tau\nu_B} \right\} . \tag{33}$$

Then the probability amplitude in Eq. (30) can be obtained as

$$|A_{q_l}(\mathbf{k}_0, \tau_B)|^2 = \frac{q_x v_1^2}{q_l \omega_B^2} |DS_l|^2 sh^2 \left(\frac{1}{2\tau\nu_B} \right) exp \left(-\frac{K_{0y} a}{\pi\tau\nu_B} \right) . \tag{34}$$

The total spontaneous emission probability is calculated from Eqs. (26) and (34) as

$$P_e^s = 2\alpha I_N(\tau\nu_B)\, \varepsilon^{1/2} \frac{L_x}{L_y} \frac{v_1^2}{c^2} \frac{\omega_c^2}{\pi^2\omega_B^2} sh^2 \left(\frac{1}{2\tau\nu_B} \right) exp \left(-\frac{K_{0y} a}{\pi\tau\nu_B} \right) \sum_{l=1}^{l_{max}} \frac{|S_l|^2}{l[1 - (\omega_c/l\omega_B)^2]^{1/2}} . \tag{35}$$

Then, it follows that for the pure harmonic miniband ($l' = 1$) the integral in Eq. (32) is reduced to the expression

$$I_l = (-1)^l \pi v_1 \frac{sh(1/2\tau\nu_B)}{[(1/2\tau\nu_B)^2 - \pi^2(l^2 - 1)] - i\pi l/\tau\nu_B} , \tag{36}$$

thus to obtain from Eq. (34) for the probability amplitude

$$|A_{q_l}(\mathbf{k}_0, \tau_B)|^2 = \pi^2 |D|^2 \frac{\omega_c v_1^2}{\omega_B^3 l} \frac{4\pi^2 sh^2(1/2\tau\nu_B)\, exp(-K_{0y}a/\pi\tau\nu_B)}{(\pi l/\tau\nu_B)^2 + [(1/2\tau\nu_B)^2 - \pi^2(l^2-1)]^2}\,; \tag{37}$$

here account has been taken for the wave vector $q_l = q_x l \omega_B/\omega_c$, and $v_1 = a\Delta_1/2\hbar$. In particular, approaching τ to infinity, we obtain the expressions characteristic for the limiting case when the scattering is totaly ignored, i.e., $I_l = -i(v_1/2)\delta_{1l}$ and

$$|A_{q_l}(\mathbf{k}_0, \tau_B)|^2 = \pi^2 |D|^2 \frac{\omega_c v_1^2}{\omega_B^3} \delta_{1l}\,. \tag{38}$$

Thus, we arrive to the conclusion that for the electron dynamics in a purely harmonic miniband when only single Fourier component of the electron velocity $v_y(\vartheta_k)$ with $l = 1$ is nonzero and in the absence of scattering ($\tau \to \infty$), all probability amplitudes $|A_{q_l}(\mathbf{k}_0, \tau_B)|^2$ are zero except the one, $l = 1$, corresponding to the fundamental Bloch frequency according to the selection rule established by Eq. (24). However, with finite τ values the higher Bloch harmonics generation becomes possible even in the pure harmonic superlattice miniband. It is seen that with increasing τ, the effect of higher Bloch harmonics generation in the pure harmonic miniband is suppressed, and for $\tau \to \infty$ it is totaly disappeared. In noting the contribution from higher harmonics of Bloch frequency into the total SE probability, note that the NNTB approximation in Eq. (31) is obtained by letting $\Delta_{l'} = \Delta_1 \delta_{1l'}$ so that $v_{l'} = v_1 \delta_{1l'}$. The occurrence of the Kronecker symbol, $\delta_{1l'}$, allows the contribution of $l' = 1$ term only whereas all the other terms are equal to zero, thereby limiting, within the NNTB approximation and without of scattering, the generation to the fundamental Bloch harmonic.

From equations (26) and (37), the total spontaneous emission probability becomes

$$P_e^s = 2\alpha I_N(\tau\nu_B)\, \varepsilon^{1/2} \frac{L_x}{L_y} \frac{v_1^2}{c^2} \frac{\omega_c^2}{\omega_B^2} \sum_{l=1}^{l_{max}} \frac{4\pi^2 sh^2(1/2\tau\nu_B)\, exp(-K_{0y}a/\pi\tau\nu_B)}{l[1-(\omega_c/l\omega_B)^2]^{1/2} \{(\pi l/\tau\nu_B)^2 + [(1/2\tau\nu_B)^2 - \pi^2(l^2-1)]^2\}}\,. \tag{39}$$

D. Total SE power estimate

It follows from Eqs. (39) that spontaneous emission probability corresponding to the fundamental Bloch harmonic ($l = 1$) is given by

$$P_e^s(l=1, \tau) = 8\pi^2 \alpha I_N(\tau\nu_B)\, \varepsilon^{1/2} \frac{L_x}{L_y} \frac{v_1^2}{c^2} \frac{\omega_c^2}{\omega_B^2} \frac{sh^2(1/2\tau\nu_B)\, exp(-K_{0y}a/\pi\tau\nu_B)}{[1-\omega_c/\omega_B)^2]^{1/2}[(\pi/\tau\nu_B)^2 + (1/2\tau\nu_B)^4]}\,. \tag{40}$$

In noting that the SE probability of Bloch radiation in the absence of scattering is given[1] by the expression

$$P_e^s(l=1,\tau\to\infty) = 2\alpha N\,\frac{L_x}{L_y}\,\frac{v_1^2}{c^2}\,\frac{\varepsilon^{1/2}\omega_c^2}{\omega_B^2(1-\omega_c^2/\omega_B^2)^{1/2}}\,, \tag{41}$$

we can compare both the probabilities for SE at fundamental Bloch frequency into the waveguide TE_{10} mode with and without the scattering taking into account analyzing the ratio

$$P_{se}^r \equiv \frac{P_e^s(l=1,\tau)}{P_e^s(l=1,\tau\to\infty)} = \frac{1}{N}\,I_N(\tau\nu_B)\,\frac{sh^2(1/2\tau\nu_B)}{(1/2\tau\nu_B)^2[1+(1/4\pi\tau\nu_B)^2]}\,; \tag{42}$$

here use has been made of Eq. (40) for the $P_e^s(l=1,\tau)$ with zero value of the electron initial wave vector component in the direction of the constant electric field. From this equation, we obtain for the limit of $\tau\to\infty$, with N finite, the expected result $P_{se}^r\to 1$. Also, for this limit, the integral (27) becomes a linear function of N, $I_N(\tau\nu_B\to\infty)=N$, and the result of Eq. (40) goes to that of Eq. (41). For finite values of the dephasing time τ, it is seen from Eqs. (40) and (41) that the linear dependence of P_e^s on N in Eqs. (41) is replaced by the factor $I_N(\tau\nu_B)$ which is a slower increasing function of N. The dependence of $I_N(\tau\nu_B)$ on N is shown in Fig. 5 for several values of the parameter $\tau\nu_B$. In Fig. 6, we show the dependence of the relative SE probability P_{se}^r [Eq.(42)] calculated as a function of number of Bloch oscillations N at different values of dephasing parameter $\tau\nu_B$. It is seen that the degrading effect of dephasing on the total SE probability within the relaxation time approximation, P_e^s, is that it becomes damped with the extent of damping dependent on the parameter $\tau\nu_B$. For numerical estimation of the damping, we assume that $\tau\nu_B=10$ and $N=10$ which corresponds to the case of reasonable scattering with the characteristic parameter $\tau\nu_B/N=1$. Then using the data of Fig. 6 (upper curve), we obtain $P_{se}^r\simeq 0.5$, that is the total SE probability, and thus the generated power of spontaneous emission, is damped approximately by a factor of 2 due to dephasing in comparison with that in the absence of scattering. Then, assuming the previously analyzed conditions[1] for the superlattice parameters, the applied electric field, and the waveguide parameters, we can use the estimated SE power output in the absence of scattering, $W\simeq 3\,\mu W$, which results in the estimation of the damped SE power generated under dephasing effects as $W\simeq 1.5\,\mu W$.

VI. SUMMARY OF THE MOST IMPORTANT RESULTS

- The quantum electron dynamics and spontaneous emission of radiation for a Bloch electron traversing a single energy miniband of a superlattice accelerating under the influence of superimposed constant external and inhomogeneous internal electric fields while radiating into a microcavity have been analyzed. The analysis is based on the use of instantaneous eigenstates of the Bloch Hamiltonian good to all orders of the dc field, and to first-order perturbation theory in the quantized radiation field.

- It is shown that the spontaneous emission for the cavity-enhanced Bloch electron probability amplitude becomes damped and frequency shifted due to the perturbing inhomogeneity when treated in a *long-time*, time-dependent perturbation theory relative to the Bloch-accelerated dynamics in the electrodynamic radiation field.

- It is found the frequency shift is proportional to the diagonal matrix elements of the Hamiltonian for the perturbing inhomogeneity with respect to the instantaneous Bloch eigenstates, and the damping term is proportional to the off-diagonal transition matrix elements of the perturbing Hamiltonian with the instantaneous eigenstates summed to the appropriate final states as determined in a *golden-rule* like fashion.

- The analysis for the special cases (slowly or abruptly varying inhomogeneities as well as a *comb* of Slater-Koster impurities with varying strength positioned at all lattice sites) showed that the frequency shift is a constant, and the damping term is slowly varying with Brillouin zone wavevector.

- The general analysis allowed to justify the *relaxation time approximation* where the damping term is considered to be a constant and the frequency shift is ignored; in this case, it is shown that the selection rules for spontaneous emission are negligibly affected, although the total probability for emission shows interesting damping and mixing characteristics.

- The spontaneous emission rate in the terahertz frequency range has been analyzed as a function of the Bloch frequency, the external electric field, number of oscillations, and dephasing parameter. The analysis of the total power shows that the dephasing degradation effects are more than compensated for by the enhancements derived by microcavity-based confinement engineering.

VII. APPENDIXES

APPENDIX A: TIME-DEPENDENT DOUBLE PERTURBATION THEORY APPROACH

The equation for the probability amplitudes $A_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t)$ in Eq. (3) is given by

$$\frac{dA_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t)}{dt} = \frac{1}{i\hbar}\sum_{\mathbf{k}'}\sum_{\{n'_{\mathbf{q},j}\}} A_{\{n'_{\mathbf{q},j}\}}(\mathbf{k}',t)\langle\{n_{\mathbf{q},j}\},\psi_{n_0\mathbf{k}(t)}|(H_I+V)|\psi_{n_0\mathbf{k}'(t)},\{n'_{\mathbf{q},j}\}\rangle$$
$$\times exp\left\{-\frac{i}{\hbar}\int_{t_0}^{t}[\varepsilon_{n_0}(\mathbf{k}'(t'))-\varepsilon_{n_0}(\mathbf{k}(t'))+\sum_{\mathbf{q},j}(n'_{\mathbf{q},j}-n_{\mathbf{q},j})\hbar\omega_q]dt'\right\}. \quad \text{(A1)}$$

We assume that at initial time, t_0, the system is in one of the eigenstates of Hamiltonian H_0+H_r with wave function $|\psi_{n_0\mathbf{K}_0},\{n^0_{\mathbf{q},j}\}\rangle$, corresponding to the Bloch electron in a single band "n_0" with the wave vector $\mathbf{K_0}$, i. e., $\psi_{n_0\mathbf{K}_0} = (1/\Omega^{1/2})\,e^{i\mathbf{K}_0\cdot\mathbf{r}}u_{n_0\mathbf{K}_0}$, and with the initial distribution of photon numbers in the radiation field $|\{n^0_{\mathbf{q},j}\}\rangle$. Substituting $A_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t) = A^{(0)}_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t) + A^{(1)}_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t) + ...$ into Eq. (A1), and taking into account the initial condition $A_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t_0) = \{\delta_{n_{\mathbf{q},j},n^0_{\mathbf{q},j}}\}\delta_{\mathbf{k}(t),\mathbf{k}_0(t)}$, one obtains to the zeroth and first order in H_I for $A^{(0)}_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t)$ and $A^{(1)}_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t)$, respectively,

$$A^{(0)}_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t) = \{\delta_{n_{\mathbf{q},j},n^0_{\mathbf{q},j}}\}\delta_{\mathbf{k}(t),\mathbf{k}_0(t)}\,, \quad \text{(A2)}$$

and

$$\frac{dA^{(1)}_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t)}{dt} - \frac{1}{i\hbar}\sum_{\mathbf{k}'}A^{(1)}_{\{n_{\mathbf{q},j}\}}(\mathbf{k}',t)V_{\mathbf{k},\mathbf{k}'}exp\left\{-\frac{i}{\hbar}\int_{t_0}^{t}[\varepsilon_{n_0}(\mathbf{k}'(t'))-\varepsilon_{n_0}(\mathbf{k}(t'))]dt'\right\}$$
$$= \frac{1}{i\hbar}\langle\{n_{\mathbf{q},j}\},\psi_{n_0\mathbf{k}(t)}|H_I|\psi_{n_0\mathbf{k}'_0(t)},\{n^0_{\mathbf{q},j}\}\rangle$$
$$\times exp\left\{-\frac{i}{\hbar}\int_{t_0}^{t}[\varepsilon_{n_0}(\mathbf{k}_0(t'))-\varepsilon_{n_0}(\mathbf{k}(t'))+\sum_{\mathbf{q},j}(n^0_{\mathbf{q},j}-n_{\mathbf{q},j})\hbar\omega_q]dt'\right\}. \quad \text{(A3)}$$

Matrix elements for perturbation operator H_I (Appendix B) are given by

$$\langle\{n_{\mathbf{q}}\},\psi_{n_0\mathbf{k}(t)}|H_I|\psi_{n_0\mathbf{k}_0(t)},\{n^0_{\mathbf{q}}\}\rangle = \sqrt{\frac{\pi c\alpha}{\omega_c\varepsilon V}}\sum_{\mathbf{q}'_z}\sum_{s=1,2}\frac{1}{[1+(q_z{}'/q_x)^2]^{1/4}}\nabla_{\mathbf{k}_y}\varepsilon_{n_0}[\mathbf{k}(t)]$$
$$\times\left[\sqrt{n^0_{\mathbf{q}'}}(\delta_{n_{\mathbf{q}'},n^0_{\mathbf{q}'}-1};\{\delta_{n_{\mathbf{q}},n^0_{\mathbf{q}}}\})\,\delta_{\mathbf{k}_0,\mathbf{k}-\mathbf{q}'_s}+\sqrt{n^0_{\mathbf{q}'}+1}(\delta_{n_{\mathbf{q}'},n^0_{\mathbf{q}'}+1};\{\delta_{n_{\mathbf{q}},n^0_{\mathbf{q}}}\})\,\delta_{\mathbf{k}_0,\mathbf{k}+\mathbf{q}'_s}\right], \quad \text{(A4)}$$

where $\alpha = e^2/\hbar c$ is the fine structure constant.

For shortness of notations, Eq. (A3) can be written as

$$\dot{A}_n(\mathbf{k},t) - \frac{1}{i\hbar}\sum_{\mathbf{k}'} f_{\mathbf{k}\mathbf{k}'} A_n(\mathbf{k}',t) = \dot{A}^0_n(\mathbf{k},t)\,, \tag{A5}$$

with

$$f_{\mathbf{k}\mathbf{k}'} = V_{\mathbf{k}\mathbf{k}'}\, exp\left\{-\frac{i}{\hbar}\int_{t_0}^{t}[\varepsilon_{n_0}(\mathbf{k}'(t')) - \varepsilon_{n_0}(\mathbf{k}(t'))]dt'\right\}\,, \tag{A6}$$

where $V_{\mathbf{k}\mathbf{k}'}$ is defined in Eq. (B10), and

$$\dot{A}^0_n(\mathbf{k},t) = \frac{1}{i\hbar}\langle\{n_{\mathbf{q},j}\},\psi_{n_0\mathbf{k}(t)}|H_I|\psi_{n_0\mathbf{k}'_0(t)},\{n^0_{\mathbf{q},j}\}\rangle$$
$$\times exp\left\{-\frac{i}{\hbar}\int_{t_0}^{t}[\varepsilon_{n_0}(\mathbf{k}_0(t')) - \varepsilon_{n_0}(\mathbf{k}(t')) + \sum_{\mathbf{q},j}(n^0_{\mathbf{q},j} - n_{\mathbf{q},j})\hbar\omega_q]dt'\right\}\,. \tag{A7}$$

Also to simplify notations, we designated $A^{(1)}_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t) \equiv A_n(\mathbf{k},t)$. Then separating terms with diagonal and off-diagonal matrix elements $f_{\mathbf{k},\mathbf{k}'}$ in Eq. (A5) so that

$$\dot{A}_n(\mathbf{k},t) - \frac{1}{i\hbar}f_{\mathbf{k}\mathbf{k}}A_n(\mathbf{k},t) - \frac{1}{i\hbar}\sum_{\mathbf{k}'\neq\mathbf{k}} f_{\mathbf{k}\mathbf{k}'} A_n(\mathbf{k}',t) = \dot{A}^0_n(\mathbf{k},t)\,, \tag{A8}$$

the equation for $A_n(\mathbf{k}',t)$ in the third term on the left-hand side of (A8) can be written in a similar fashion

$$\dot{A}_n(\mathbf{k}',t) - \frac{1}{i\hbar}f_{\mathbf{k}'\mathbf{k}}A_n(\mathbf{k},t) - \frac{1}{i\hbar}\sum_{\mathbf{k}''\neq\mathbf{k}} f_{\mathbf{k}'\mathbf{k}''} A_n(\mathbf{k}'',t) = \dot{A}^0_n(\mathbf{k}',t)\,. \tag{A9}$$

Following to the Wigner-Weiskopf-like approximation, we ignore the third term on the right-hand side of (A9); then, after integration on time, we find

$$A_n(\mathbf{k}',t) = \frac{1}{i\hbar}\int_{t_0}^{t} dt'\, f_{\mathbf{k}'\mathbf{k}}(t')A_n(\mathbf{k},t') + A^0_n(\mathbf{k}',t)\,, \tag{A10}$$

where we have taken into account that $A_n(\mathbf{k}',t_0) = \delta_{\mathbf{k}',\mathbf{k}_0}$, i.e., is equal to zero for $\mathbf{k}' \neq \mathbf{k}_0$, and

$$A^0_n(\mathbf{k},t) = \int_{t_0}^{t} \dot{A}^0_n(\mathbf{k},t')dt'\,. \tag{A11}$$

Thus, by making use of (A10) in Eq. (A8), we obtain the latter equation in the form

$$\dot{A}_n(\mathbf{k},t) - \frac{1}{i\hbar}f_{\mathbf{k}\mathbf{k}}A_n(\mathbf{k},t) + \frac{1}{\hbar^2}\sum_{\mathbf{k}'\neq\mathbf{k}} f_{\mathbf{k}\mathbf{k}'}(t)\int_{t_0}^{t} dt'\, f^*_{\mathbf{k}\mathbf{k}'}(t')A_n(\mathbf{k},t')$$
$$= \dot{A}^0_n(\mathbf{k},t) + \frac{1}{i\hbar}\sum_{\mathbf{k}'\neq\mathbf{k}} f_{\mathbf{k}\mathbf{k}'}(t)A^0_n(\mathbf{k}',t)\,. \tag{A12}$$

APPENDIX B: MATRIX ELEMENTS OF PERTURBATION HAMILTONIAN

Matrix elements of perturbation Hamiltonian H_I for the perturbing quantum radiation field are calculated by substituting H_I given in Sec. II, with the vector potential $\mathbf{A}_r$ of Eq. (2), and the wave function $\psi_{n_0\mathbf{k}(t)}$; then we obtain

$$\langle\{n_{\mathbf{q}}\},\psi_{n_0\mathbf{k}(t)}|H_I|\psi_{n_0\mathbf{k}'(t)},\{n'_{\mathbf{q}}\}\rangle = -\frac{e}{m_0c}\sqrt{\frac{4\pi\hbar c^2}{\omega_c\varepsilon V}}\sum_{\mathbf{q}'_z}\frac{1}{[1+(q_z{}'/q_x)^2]^{1/4}}$$
$$\times\left[\sqrt{n'_{\mathbf{q}'}}(\delta_{n_{\mathbf{q}'},n'_{\mathbf{q}'}-1};\{\delta_{n_{\mathbf{q}},n'_{\mathbf{q}}}\})\,I^{(+)}_{\mathbf{k},\mathbf{k}'}+\sqrt{n'_{\mathbf{q}'}+1}(\delta_{n_{\mathbf{q}'},n'_{\mathbf{q}'}+1};\{\delta_{n_{\mathbf{q}},n'_{\mathbf{q}}}\})\,I^{(-)}_{\mathbf{k},\mathbf{k}'}\right], \tag{B1}$$

where the first term on the right-hand side of Eq. (B1) is due to absorption of a photon from the radiation field by the Bloch electron and the second term represents emission of a photon by the Bloch electron to the radiation field; also we denoted $(\delta_{n_{\mathbf{q}'},n'_{\mathbf{q}'}\pm1};\{\delta_{n_{\mathbf{q}},n'_{\mathbf{q}}}\})\equiv \delta_{n_{\mathbf{q}'},n'_{\mathbf{q}'}\pm1}\prod_{\mathbf{q}\neq\mathbf{q}'}\delta_{n_{\mathbf{q}},n'_{\mathbf{q}}}$ and

$$I^{(\pm)}_{\mathbf{k},\mathbf{k}'}=\int\psi^*_{n_0\mathbf{k}}\cos(q_xx)(\mathbf{p}+\mathbf{p}_c)_y e^{\pm iq'_zz}\psi_{n_0\mathbf{k}'}d\mathbf{r}\,, \tag{B2}$$

where $(\mathbf{p}+\mathbf{p}_c)_y$ is the y component of $[\mathbf{p}+\mathbf{p}_c(t)]$. In using $\psi_{n_0\mathbf{k}(t)}(\mathbf{r},t)=\Omega^{-1/2}e^{i\mathbf{K}\cdot\mathbf{r}}u_{n_0\mathbf{k}(t)}(\mathbf{r},t)$, the integrals $I^{(\pm)}_{\mathbf{k},\mathbf{k}'}$ are equal to zero except when $\mathbf{K}'-\mathbf{K}\pm\mathbf{q}_s=\mathbf{G}$, where $\mathbf{G}$ is the vector of reciprocal lattice and $\mathbf{q}_s=\{\pm q_x,0,q_z\}$ with " + " for $s=1$ and " − " for $s=2$ (Here we omit, for brevity, the prime on the $\mathbf{q}$ vector). For values of $\mathbf{K}$ and $\mathbf{q}_s$ in the first Brillouin zone, we can take $\mathbf{G}=0$. Then, it follows that $\mathbf{K}'=\mathbf{K}\mp\mathbf{q}_s$, and since $\mathbf{k}(t)=\mathbf{K}+\mathbf{p}_c(t)/\hbar$, then we get $\mathbf{k}'=\mathbf{k}\mp\mathbf{q}_s$. In this case, the integrals $I^{(\pm)}_{\mathbf{k},\mathbf{k}'}$ are obtained as

$$I^{(\pm)}_{\mathbf{k},\mathbf{k}'}=\frac{1}{2}\sum_{s=1,2}\mathcal{I}^{\pm}_s\delta_{\mathbf{k}',\mathbf{k}\mp\mathbf{q}_s}\,, \tag{B3}$$

with

$$\mathcal{I}^{(\pm)}_s=\frac{1}{\Omega_0}\int(e^{i\mathbf{K}\mathbf{r}_0}u_{n_0\mathbf{k}})^*(\mathbf{p}+\mathbf{p}_c)_y e^{i\mathbf{K}\mathbf{r}_0}u_{n_0\mathbf{k}\mp\mathbf{q}_s}d\mathbf{r}_0\,, \tag{B4}$$

where the integration over $d\mathbf{r}_0$ is carried out over the primitive cell volume Ω_0. In the long-wavelength limit ($qa\ll1$), the integrals $\mathcal{I}^{(\pm)}_s$ can be approximated as

$$\mathcal{I}^{(\pm)}_s=\frac{1}{\Omega_0}\int(e^{i\mathbf{K}\mathbf{r}_0}u_{n_0\mathbf{k}})^*(\mathbf{p}+\mathbf{p}_c)_y e^{i\mathbf{K}\mathbf{r}_0}u_{n_0\mathbf{k}}d\mathbf{r}_0=\int\psi^*_{n_0\mathbf{k}}(\mathbf{p}+\mathbf{p}_c)_y\psi_{n_0\mathbf{k}}d\mathbf{r}\,. \tag{B5}$$

In noting that the well-known momentum[8] expectation value for Bloch states is

$$\int(e^{i\mathbf{k}(t)\mathbf{r}}u_{n_0\mathbf{k}})^*\mathbf{p}\,(e^{i\mathbf{k}(t)\mathbf{r}}u_{n_0\mathbf{k}})d\mathbf{r}=m_0\mathbf{v}[\mathbf{k}(t)]\,, \tag{B6}$$

where $\mathbf{v}[\mathbf{k}(t)] = (1/\hbar)\nabla_{\mathbf{K}}\varepsilon_{n_0}(\mathbf{K})|_{\mathbf{k}(t)}$, it then follows, for $\mathbf{k}(t) = \mathbf{K} + \mathbf{p}_c(t)/\hbar$ in Eq. (B6), that

$$\int \psi^*_{n_0\mathbf{k}}(\mathbf{p} + \mathbf{p}_c)\,\psi_{n_0\mathbf{k}}d\mathbf{r} = m_0\mathbf{v}(\mathbf{k})\,. \tag{B7}$$

Thus, Eqs. (B3) - (B7) become

$$I^{(\pm)}_{\mathbf{k},\mathbf{k}'} = \frac{1}{2}m_0 v_y(\mathbf{k})\sum_{s=1,2}\delta_{\mathbf{k}',\mathbf{k}\mp\mathbf{q}_s}\,. \tag{B8}$$

Then, using (B1) and (B8), the matrix elements of H_I are established for use in Eq. (A4).

Matrix elements of the perturbing inhomogeneity V are diagonal with respect to the photon quantum numbers

$$\langle\{n_{\mathbf{q}}\},\psi_{n_0\mathbf{k}(t)}|V|\psi_{n_0\mathbf{k}'(t)},\{n^{'}_{\mathbf{q}}\}\rangle = \{\delta_{n_{\mathbf{q}},n'_{\mathbf{q}}}\}V_{\mathbf{k}\mathbf{k}'}\,, \tag{B9}$$

where we denoted for the matrix elements of V between the instantaneous eigenstates

$$V_{\mathbf{k}\mathbf{k}'} = (\psi_{n_0\mathbf{k}(t)}, V\psi_{n_0\mathbf{k}'(t)}) = \int \psi^*_{n_0\mathbf{k}(t)}V(\mathbf{r},t)\psi_{n_0\mathbf{k}'(t)}d\mathbf{r}\,. \tag{B10}$$

[1] V. N. Sokolov, G. J. Iafrate, and J. B. Krieger, Phys. Rev. B **75**, 045330 (2007).

[2] L. Esaki and R. Tsu, IBM J. Res. Dev. **14**, 61 (1970).

[3] J. B. Krieger and G. J. Iafrate, Phys. Rev. B **33**, 5494 (1986); G. J. Iafrate and J. B. Krieger, *ibid.* **40**, 6144 (1989).

[4] N. Marcuvitz, *Waveguide Handbook* (Peregrinus, London, 1993).

[5] W. H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973), p. 57.

[6] E. Merzbacher, *Quantum Mechanics* (Wiley, New York, 1970).

[7] V. N. Sokolov, G. J. Iafrate, and J. B. Krieger, *Spontaneous emission of Bloch oscillation radiation enhanced by a microcavity environment and degraded by dephasing from an inhomogeneous electric field*, paper manuscript submitted to Phys. Rev. B (2007).

[8] C. Kittel, *Quantum Theory of Solids* (Wiley and Sons, copyright: 1963, 1987), 2nd revised printing, ISBN 0-471-62412-8, p. 185, Eq. (49) therein.

[9] V. N. Sokolov, L. Zhou, G. J. Iafrate, and J. B. Krieger, Phys. Rev. B **73**, 205304 (2006).

[10] Note that **q** is not collinear with the direction of the applied field **E**, then the integral in Eq. (21) does not vanish because of the periodicity of $\varepsilon_{n_0}(\mathbf{k}_0)$, and contributes to the selection rule. For $\omega_q = \omega_c[1 + (q_z/q_x)^2]^{1/2}$, this corresponds to neglecting a small term $\sim v_\perp/c$ as compared to unity, where $v_\perp$ is the electron velocity perpendicular to the y axis.

Accomplishments from this research

1. V. N. Sokolov, G. J. Iafrate, and J. B. Krieger, *Microcavity enhancement of spontaneous emission for Bloch oscillations*, Phys. Rev. B **75**, 045330-1-6 (2007).

2. V. N. Sokolov, G. J. Iafrate, and J. B. Krieger, *Spontaneous emission of Bloch oscillation radiation enhanced by a microcavity environment and degraded by dephasing from an inhomogeneous electric field*, paper manuscript is in process.

FIGURES

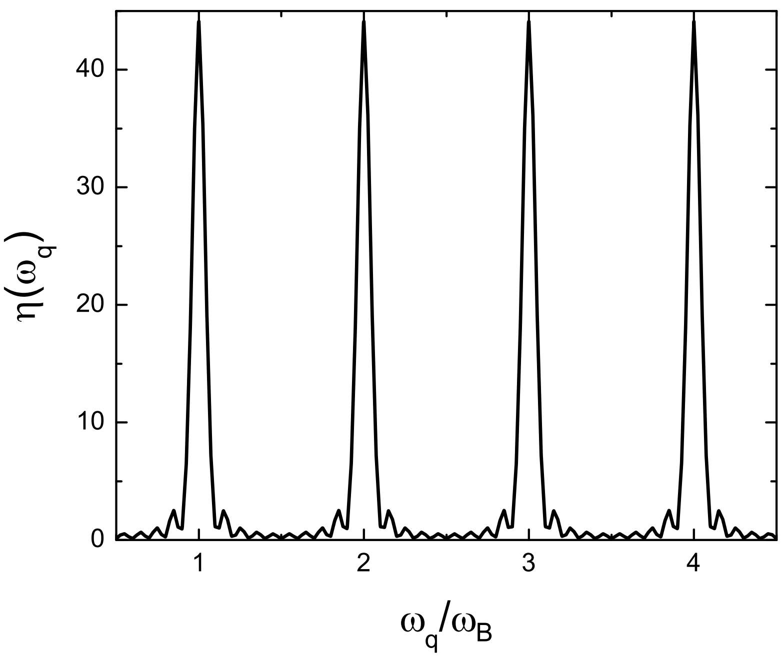

FIG. 1: Dependence of the relative probability spectral density of spontaneous emission $\eta(\omega_q)$ [transfer function of Eq. (23)] on the normalized photon frequency ω_q/ω_B calculated at resonable scattering $\tau\nu_B = 10$ for $N = 10$.

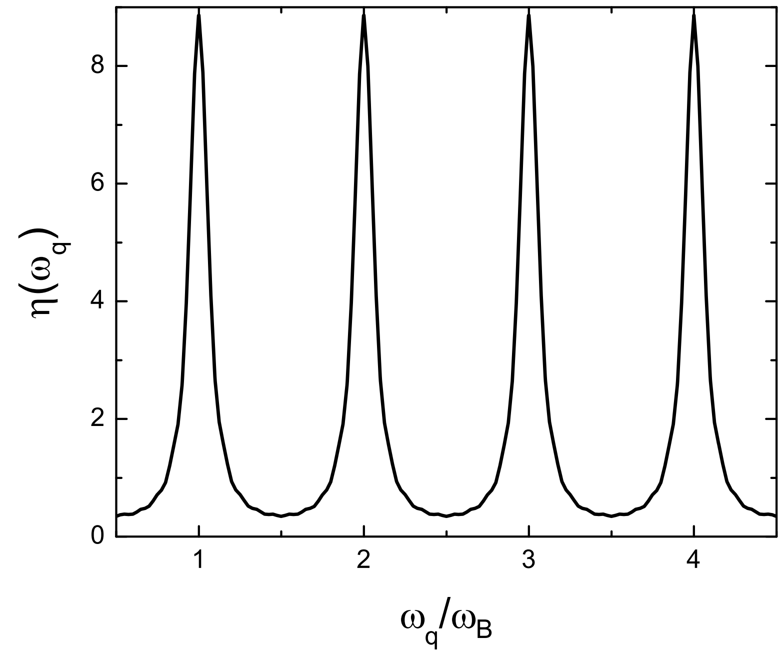

FIG. 2: Dependence of the relative probability spectral density of spontaneous emission $\eta(\omega_q)$ [transfer function of Eq. (23)] on the normalized photon frequency ω_q/ω_B calculated at strong scattering $\tau\nu_B = 2.5$ for $N = 10$.

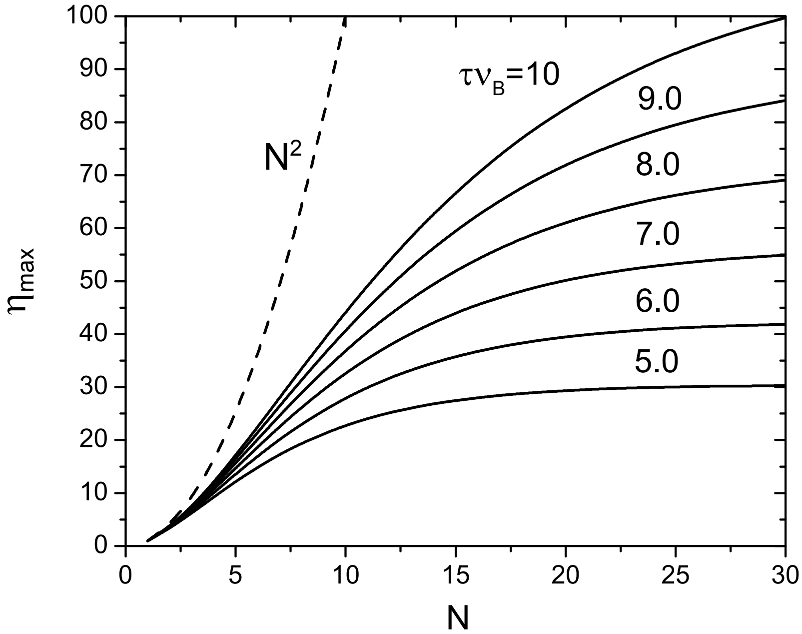

FIG. 3: Dependence of the transfer function η_{max} calculated at maximum growth conditions of Eq.(24) as a function of number of Bloch oscillations N at different values of dephasing parameter $\tau\nu_B$ [$5.0 \leq \tau\nu_B \leq 10.0$]. The dashed curve shows the dependence of $\eta_{max} \sim N^2$ corresponding to $\tau\nu_B \to \infty$, i.e., in the absence of dephasing effects.

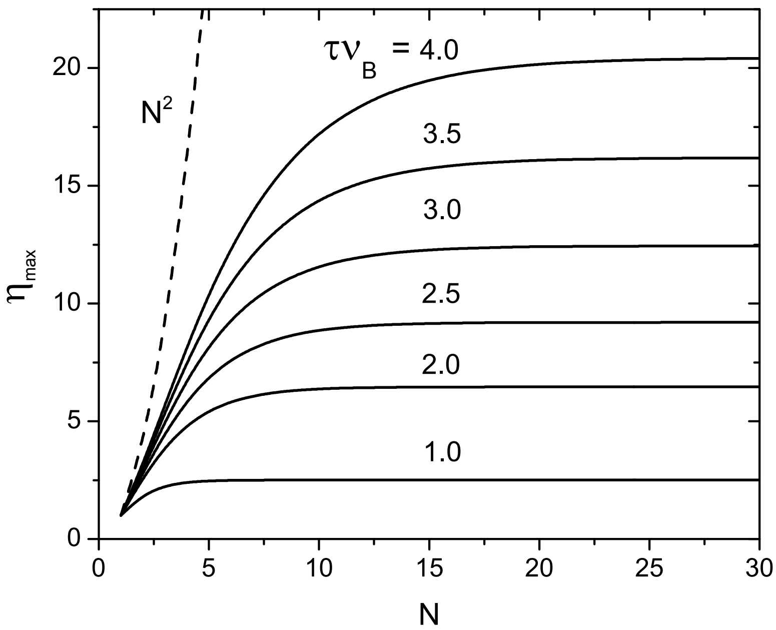

FIG. 4: Dependence of the transfer function η_{max} calculated at maximum growth conditions of Eq.(24) as a function of number of Bloch oscillations N at different values of dephasing parameter $\tau\nu_B$ [$1.0 \leq \tau\nu_B \leq 4.0$]. The dashed curve shows the dependence of $\eta_{max} \sim N^2$ corresponding to $\tau\nu_B \to \infty$, i.e., in the absence of dephasing effects.

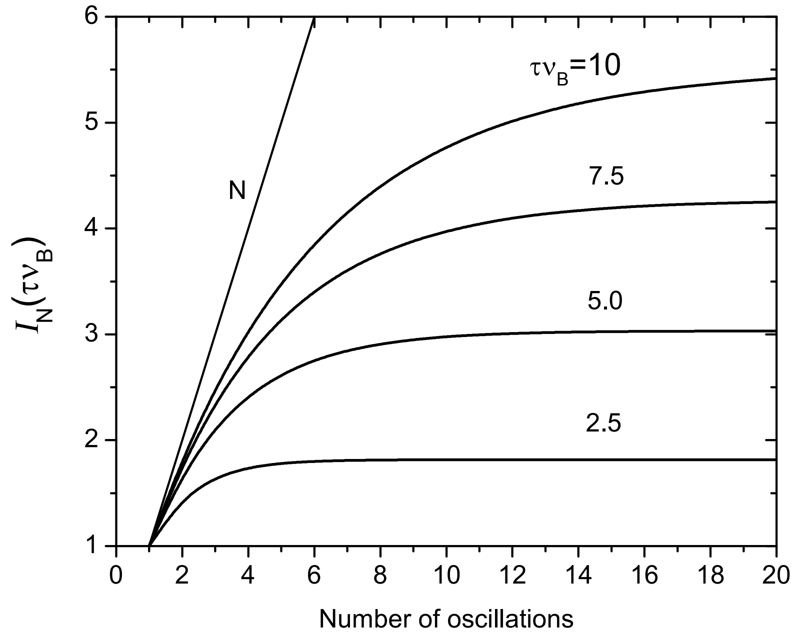

FIG. 5: Dependence of the prefactor $I_N(\tau\nu_B)$ in expression for the relative spontaneous emission probability at fundamental Bloch frequency into the waveguide TE_{10} mode P^r_{se} [Eq.(42)] calculated as a function of number of Bloch oscillations N at different values of dephasing parameter $\tau\nu_B$. The straight line shows the dependence of $I_N(\tau\nu_B) \sim N$ corresponding to $\tau\nu_B \to \infty$, i.e., in the absence of dephasing effects.

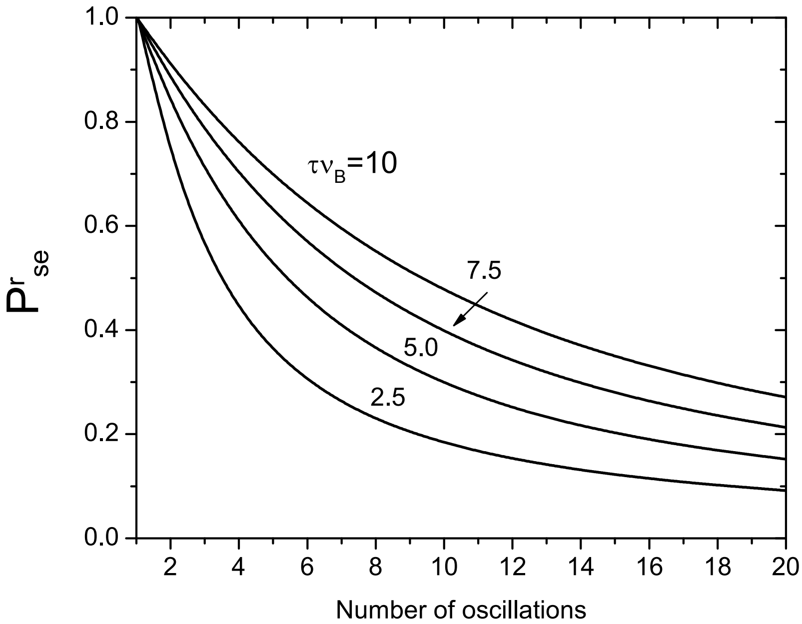

FIG. 6: Dependence of the relative spontaneous emission probability P^r_{se} [Eq.(42)] at fundamental Bloch frequency into the waveguide TE_{10} mode calculated as a function of number of Bloch oscillations N at different values of dephasing parameter $\tau\nu_B$. $P^r_{se} = 1$ $(N > 1)$ corresponds to $\tau\nu_B \to \infty$, i.e., the absence of dephasing effects.

Microcavity enhancement of spontaneous emission for Bloch oscillations

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A theory for the spontaneous emission of a Bloch electron traversing a single energy miniband of a superlattice while accelerating under the influence of a constant external electric field and radiating into a microcavity is presented. In the analysis, the quantum electromagnetic radiation field is described by the dominant microcavity TE_{10} rectangular waveguide mode in the Coulomb gauge, and the instantaneous eigenstates of the Bloch Hamiltonian are utilized as the basis states in describing the Bloch electron dynamics to all orders in the constant external electric field. The results show that the spontaneous emission amplitude, when analyzed over many integral multiple values of the Bloch period, gives rise to selection rules for photon emission in both frequency and wave number with preferred transitions at the Wannier-Stark ladder levels. From these selection rules, the total spontaneous emission probability is derived to first-order perturbation theory in the quantized radiation field. It is shown that the power radiated into the dominant TE_{10} waveguide mode can be enhanced by an *order of magnitude* over the free-space value by tuning the Bloch frequency to align with the waveguide spectral density peak. A general expression for the total spontaneous emission probability is obtained in terms of arbitrary superlattice single band parameters, showing multiharmonic behavior and cavity tuning properties. For GaAs-based superlattices, described in the nearest-neighbor tight-binding approximation, the power radiated into the waveguide from spontaneous emission due to Bloch oscillations in the terahertz frequency range is estimated to be several microwatts.

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I. INTRODUCTION

A theory of spontaneous emission (SE), including the probability for absorption, for a Bloch electron traversing a single energy band in an external constant electric field has been examined recently by the authors and colleagues.[1] The theoretical analysis was fully quantum-mechanical, in that the radiation field was described as the *free-space* quantized electromagnetic field in the Coulomb gauge. Analysis of the probability amplitude, over integral multiples of the Bloch period, resulted in *selection rules* for photon emission in both photon frequency and wave vector, showing preferred transitions to the *Wannier-Stark ladder* levels. Using these selection rules, total SE probability was derived to first-order perturbation theory in the quantized radiation field. Although the output frequency of the radiation could be operationally tuned to span the gigahertz to terahertz spectral range by appropriately fixing the constant electric field, the power output for terahertz emission into free space for a GaAs-based superlattice (SL) was estimated to be about one-tenth of a microwatt.

In this paper, we analyze the SE of radiation emitted by a Bloch electron accelerated through the single miniband of a SL structure, but now with the SL placed in a resonant microcavity. The intent of the resonant microcavity is well known,[2,3] to redistribute the free-space modal spectral density so as to increase its value at some frequencies and to decrease it at others; therefore for an active medium, such as a radiating SL placed in a cavity structure, the SE rate can be enhanced or diminished depending upon the tuning of the emission frequency relative to the cavity mode spectral density peak. We use this tuning property offered by microcavities to increase the power output for enhanced SE from GaAs-based SLs. As examples relevant to the principle idea of this work, many efforts have been focused on increasing of the SE rate in optical microcavities.[4] An increase of the overall emitted THz power of more than one order of magnitude has been reported by placing a surface-field emitter inside a THz cavity.[5]

II. QUANTUM DYNAMICS AND THEORETICAL APPROACH

For purposes of the calculation, we assume that the SL structure is placed in a waveguide with rectangular cross section $L_x \times L_y$ and length L_z, where the coordinate axes are chosen to be along the waveguide edges. The constant or dc electric field $\mathbf{E}$ is applied along the y axis, which is also the SL growth direction. The electromagnetic field inside the waveguide, with assumed perfectly conducting walls, is determined by the guided modes corresponding to standing waves with respect to the X and Y axes [designated by an integer pair (m,n)], and propagating waves along the Z axis characterized by propagation constant q_z. Such modes form a complete and orthogonal basis set for describing the electromagnetic field within the waveguide. In the following, we will consider only transverse electric (TE) modes, where the electric field is perpendicular to the direction of propagation. For practical cases,[6] the most important of all confined modes in this waveguide configuration is the TE_{10} mode ($m=1$, $n=0$), which is the dominant mode of a waveguide with $L_x > L_y$. This mode gives rise to the lowest attenuation, and the corresponding electric field $\mathbf{E}_r$, for the chosen system geometry, is polarized in the direction of the dc field $\mathbf{E}$. Therefore we consider only one excited, TE_{10}, mode, and ignore all the other less effective TE and TM modes. For the remaining TE_{10} mode, the following conditions on $\mathbf{E}_r$ and $\mathbf{H}_r$ are in effect, namely that $E_{r,x}=E_{r,z}=0$ and magnetic field component $H_{r,y}=0$. Therefore it follows that the vector potential $\mathbf{A}_r$ for the waveguide field has only one nonzero (y) component, namely,[7]

$$A_{r,y}=\sum_{q_z}\sqrt{\frac{4\pi\hbar c^2}{\omega_q\varepsilon V}}\sin(q_x x)(\hat{a}_q e^{iq_z z}+\hat{a}_q^\dagger e^{-iq_z z}),\qquad(1)$$

where $\hat{a}_q^\dagger$ and $\hat{a}_q$ are the photon boson creation and annihilation operators, $q_x=\pi/L_x$, c is the velocity of light in vacuum, $V=L_xL_yL_z$, and ε is the dielectric constant of the medium filling the waveguide. The normalization constant in Eq. (1) is chosen in such a way that the Hamiltonian for the quantized radiation field has the form $H_r=\Sigma_q\hbar\omega_q\hat{a}_q^\dagger\hat{a}_q$, where $\omega_q=\omega_c[1+(q_z/q_x)^2]^{1/2}$ is the mode dispersion relation, and $\omega_c=q_xc/\sqrt{\varepsilon}$ is the angular cutoff frequency determined by the waveguide geometry. The guided mode wavelength is written as $\lambda=\lambda_c/[(\omega_q/\omega_c)^2-1]^{1/2}$, where $\lambda_c=2L_x$ is the cutoff wavelength.[8]

The Bloch dynamical properties are now considered for the situation in which the electron is confined to a single miniband, "n_0," of a SL with band energy $\varepsilon_{n_0}(\mathbf{K})$, while the *effects of interband coupling*[9] *and electron intraband scattering are ignored*. Therefore the quantum dynamics is described by the time-dependent Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}|\Psi_{n_0}(t)\rangle=H|\Psi_{n_0}(t)\rangle,\qquad(2)$$

where the exact Hamiltonian $H=[\mathbf{p}-(e/c)\mathbf{A}]^2/2m_0+V_c(\mathbf{r})+H_r$ can be reduced to a sum of the following separate Hamiltonians with $H=H_0+H_r+H_I$.[1] Here the first two terms represent the Hamiltonian $H_0(t)=[\mathbf{p}+\mathbf{p}_c(t)]^2/2m_0+V_c(\mathbf{r})$, for a single electron in a periodic crystal potential, $V_c(\mathbf{r})$, interacting with a homogeneous electric field, and the Hamiltonian H_r for the cavity mode electromagnetic radiation field. The total vector potential in the exact Hamiltonian consists of $\mathbf{A}=\mathbf{A}_c+\mathbf{A}_r$, where $\mathbf{A}_c=-(c/e)\mathbf{p}_c$ describes the external electric field with $\mathbf{p}_c(t)=e\int_{t_0}^t\mathbf{E}(t')dt'$, and $\mathbf{A}_r$, given in Eq. (1), describes the cavity mode quantized radiation field; also, m_0 is the free-electron mass. The Hamiltonian $H_I(t)=-(e/m_0c)\mathbf{A}_r\cdot[\mathbf{p}+\mathbf{p}_c(t)]$, for the first-order interaction between the quantum field and the Bloch electron, couples both subsystems H_0 and H_r, and causes transitions between the accelerated Bloch electron states through photon absorption and emission. Then, starting with the reduced Hamiltonian $H=H_0+H_r+H_I$, use is made of first-order time-dependent perturbation theory to calculate SE transitions probabilities between states of H_0+H_r while regarding $H_I(t)\sim\mathbf{A}_r\cdot[\mathbf{p}+\mathbf{p}_c(t)]$ as a perturbation.[10] The solution to $|\Psi_{n_0}(t)\rangle$ of Eq. (2) can be represented in terms of eigenstates of basis states $|\psi_{n_0\mathbf{k}(t)},\{n_{\mathbf{q},j}\}\rangle=|\psi_{n_0\mathbf{k}(t)}\rangle|\{n_{\mathbf{q},j}\}\rangle$ of the unperturbed Hamiltonian H_0+H_r as

$$|\Psi_{n_0}(t)\rangle=\sum_{\mathbf{k}}\sum_{\{n_{\mathbf{q},j}\}}A_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t)|\psi_{n_0\mathbf{k}(t)},\{n_{\mathbf{q},j}\}\rangle\times\exp\left\{-\frac{i}{\hbar}\int_{t_0}^t\left[\varepsilon_{n_0}(\mathbf{k}(t'))+\sum_{\mathbf{q},j}\hbar\omega_q n_{\mathbf{q},j}\right]dt'\right\},\qquad(3)$$

where the summation over $\mathbf{k}$ is carried out over the entire Brillouin zone, and $\{n_{\mathbf{q},j}\}$ is specified over all possible combinations of photon occupation number $n_{\mathbf{q},j}$ with photon wave vectors $\mathbf{q}$ and certain polarization $(j=1,2)$. The instantaneous eigenstates of H_0 are given by[9] $\psi_{n_0\mathbf{k}(t)}(\mathbf{r},t)=\Omega^{-1/2}e^{i\mathbf{K}\cdot\mathbf{r}}u_{n_0\mathbf{k}(t)}(\mathbf{r},t)$, where $u_{n_0\mathbf{k}(t)}(\mathbf{r},t)$ is the periodic part of the Bloch function, $\mathbf{k}(t)=\mathbf{K}+\mathbf{p}_c(t)/\hbar$, and the values of the electron wave vector $\mathbf{K}$ are determined by the periodic boundary conditions of the periodic crystal of volume Ω.

III. PROBABILITY AMPLITUDES, SELECTION RULES, AND TOTAL SPONTANEOUS EMISSION PROBABILITY

A. Probability amplitudes—one-photon emission

For the case of one-photon SE, which assumes that initially no photons are present in the radiation field, the probability amplitude in the wave function of Eq. (3) satisfies the initial condition $A_{\{n_{\mathbf{q},j}\}}(\mathbf{k},t_0)=\{\delta_{n_{\mathbf{q},j},0}\}\delta_{\mathbf{K},\mathbf{K}_0}$ at time $t=t_0$ when the external electric field is turned on. Here, $\mathbf{K}_0$ and $n^0_{\mathbf{q},j}=0$ are the initial values of $\mathbf{K}$ and $n_{\mathbf{q},j}$. The probability amplitude for SE, $A_q(\mathbf{k}_0,t)$, *at any time t*, is now evaluated in first-order perturbation theory[1] as

$$A_q(\mathbf{k}_0,t)=D(q_x/q)^{1/2}\int_{t_0}^t dt' v_y(\mathbf{k}_0-\mathbf{q}_s)\times\exp\left\{-\frac{i}{\hbar}\int_{t_0}^{t'}[\varepsilon_{n_0}(\mathbf{k}_0)-\varepsilon_{n_0}(\mathbf{k}_0-\mathbf{q}_s)-\hbar\omega_q]dt_1\right\},\qquad(4)$$

where $D=-i\sqrt{\pi c\alpha/\omega_c\varepsilon V}$, $\alpha=e^2/\hbar c$ is the fine-structure constant, $\mathbf{k}_0(t)=\mathbf{K}_0+\mathbf{p}_c(t)/\hbar$, $\mathbf{q}_s=\{\pm q_x,0,q_z\}$ with the "+" used for $s=1$ and the "−" used for $s=2$, $v_y(\mathbf{k}(t))=(1/\hbar)\nabla_{\mathbf{K_y}}\varepsilon_{n_0}(\mathbf{K})|_{\mathbf{k}(t)}$, the y component of Bloch velocity in the band, and $q=(q_x^2+q_z^2)^{1/2}$. Then the spontaneous emission process results in the *total SE probability at a time t*, $P_e^s(t)$, given by

$$P_e^s(t)=\sum_q\sum_{s=1,2}|A_q(\mathbf{k}_0,t)|^2.\qquad(5)$$

In evaluating $A_q(\mathbf{k}_0,t)$, we take into account that the external dc field $\mathbf{E}$ is along the Y axis; then, it follows that $k_{0y}(t)=K_{0y}+eE(t-t_0)/\hbar$. In taking advantage of the *periodic* properties of the terms in Eq. (4), $A_q(\mathbf{k}_0,t)$ is easily evaluated in terms of *clocked* integral multiples of the Bloch period; here, $t=N\tau_B$, where $\tau_B=2\pi/\omega_B$, the time to traverse one period of the Brillouin zone. Then the time integral in Eq. (4) can be replaced by an integral over k_{0y} through the substitution $dt=(\hbar/eE)dk_{0y}$, and the probability amplitude, at integral multiples of the Bloch period, can be expressed in terms of an integral over the single Bloch period, τ_B.[1,9] Thus we obtain

$$|A_q(\mathbf{k}_0,N\tau_B)|^2 = \frac{\sin^2(N\beta_q/2)}{\sin^2(\beta_q/2)}|A_q(\mathbf{k}_0,\tau_B)|^2, \tag{6}$$

where the parameter β_q is given by

$$\beta_q = 2\pi\frac{\omega_q}{\omega_B} + \frac{1}{eE}\int_{K_{0y}}^{K_{0y}+G_y} dk_{0y}[\varepsilon_{n_0}(\mathbf{k}_0) - \varepsilon_{n_0}(\mathbf{k}_0 - \mathbf{q}_s)], \tag{7}$$

and $G_y=2\pi/a$, the y component of the SL reciprocal-lattice vector.

B. Selection rules

From Eq. (6), it is seen that the quantity $|A_q(\mathbf{k}_0,N\tau_B)|^2$ will reach its maximum growth value when $\beta_q=2\pi(m+\delta)$, where m is a nonzero integer and $\delta\to 0$; for this limit, the function $\eta(\omega_q)=\sin^2(N\beta_q/2)/\sin^2(\beta_q/2)\to N^2$, i.e., it becomes sharply peaked at the resonances $\beta_q=2\pi m$ with increasing N. It is clear that this condition for maximum growth establishes the *selection rule*,[1,9] *for both the photon emission frequency*, ω_q, *and the key wave-vector component*, q_z. Indeed, from the condition $\beta_q=2\pi m$, it follows from Eq. (7) in the radiative long-wavelength limit ($qa\ll 1$) that one generally[11] has

$$\omega_q = m\omega_B, \quad q_z = \pm q_{zm}, \quad q_{zm} \equiv q_x\left[\left(m\frac{\omega_B}{\omega_c}\right)^2 - 1\right]^{1/2}; \tag{8}$$

this gives two conditions, the "*Wannier-Stark ladder*" resonance frequency condition, and the sustaining wave-vector cavity resonance condition. These quantization conditions are obtained, naturally, through the use of instantaneous eigenstates of the Bloch Hamiltonian without requiring any *ad hoc* assumptions concerning the existence of Wannier-Stark energy states. It is worth emphasizing that Eq. (6) should be considered simultaneously with Eq. (8), i.e., both selection rule conditions for the transition of photon frequency and wave vector should be taken into account explicitly in evaluating the total SE probability spectra given by $|A_q(\mathbf{k}_0,\tau_B)|^2$. In this regard, we note that although Eq. (6) predicts the equal relative probabilities calculated at times $t=\tau_B$ and $t=N\tau_B$ for different harmonics of Bloch frequency through

$$\frac{|A_q(\mathbf{k}_0,N\tau_B)|^2}{|A_q(\mathbf{k}_0,\tau_B)|^2} = \eta(\omega_q = m\omega_B) = N^2, \tag{9}$$

it is also clear from Eqs. (4) and (8) explicitly that the probability amplitude depends on ω_B and q independently. Therefore the *total SE probability* in Eq. (5) must incorporate, and will reflect, the importance of both selection rules.

In summary, it is noted here that the probability amplitude of Eq. (4) is markedly influenced by the time-dependent exponential phase factor. This phase factor measures the electron's field-dependent energy change from state $\varepsilon_{n_0}[\mathbf{k}_0(t)]$ to state $\varepsilon_{n_0}[\mathbf{k}_0(t)-\mathbf{q}_s]$, while simultaneously emitting a single photon $\hbar\omega_q$ to the initially prepared vacuum state of the radiation field. This form of the probability amplitude *does not* follow the usual *golden rule* for *stationary* initial-final state dependence because the instantaneous Bloch eigenstates as well as the Hamiltonian describing the external electric field based on the vector potential gauge are explicitly time dependent. Therefore as an equivalent alternative to the *golden rule*, use is made of the periodic nature of the energy band under consideration, and the maximum growth condition for the probability amplitude is established at integral multiples of the Bloch period, $N\tau_B$ with N large. Thus maximum growth in probability amplitude places a quantization condition on β_q in Eq. (7), namely, that $\beta_q=2\pi m$, which results in the selection rules of Eq. (8) for ω_q and q_z, and allows for the calculation of the total spontaneous emission probability.

C. Total spontaneous emission probability

The total SE probability is evaluated at time $t=N\tau_B$, $P_e^s=P_e^s(N\tau_B)$, by substituting $|A_q(\mathbf{k}_0,N\tau_B)|^2$ from Eq. (6) into Eq. (5), and then summing over q. The sum over q in Eq. (5) has been replaced by an integral over q, taking into account the TE_{10} mode density of states and polarization such that $\Sigma_q(\cdots)\to(L_z/\pi)\int dq(\cdots)/[1-(q_x/q)^2]^{1/2}$. The integral can be evaluated by using the property of the integrand which contains a sharply peaked, symmetric function of q at $q=q_m=(q_x^2+q_{zm}^2)^{1/2}$ [see Eq. (8)]. Thus at every node defined by the resonance conditions, the slowly varying function of q in the integrand can be replaced by its value evaluated at $q=q_m$, and then removed from the integral over q; after that, the remaining integral can be evaluated to obtain

$$P_e^s = N\frac{L_z}{L_x}\frac{\omega_B}{\omega_c}\sum_{l=1}^{l_{max}}\sum_{s=1}^{2}\frac{|A_{q_l}(\mathbf{k}_0,\tau_B)|^2}{[1-(q_x/q_l)^2]^{1/2}}. \tag{10}$$

Here l_{max} follows from $q_{max}=l_{max}(\omega_B/\omega_c)q_x$, and determines the upper limit in the sum over higher Bloch oscillation harmonics.

The calculation of P_e^s in Eq. (10) requires the use of $A_q(\mathbf{k}_0,t)$ in Eq. (4), evaluated when $t=t_0+\tau_B$,[12] and at the selection rules of Eq. (8), that is, when $\hbar\omega_q=m\hbar\omega_B$ and $q=q_m$. In addition, the dependence upon $\mathbf{q}$ in Eq. (4) is made explicit by invoking the assumption of photon long-wavelength limit, which is valid for all periodic potentials of interest, even SLs, where $q\ll\pi/a$. Thus it follows from Eq. (4) that

$$|A_{q_l}(\mathbf{k}_0,\tau_B)|^2 = \frac{q_x}{q_l(\omega_B/2\pi)^2}|DI_l|^2, \tag{11}$$

where

$$I_l = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\vartheta_k v_y(\vartheta_k)\exp(-il\vartheta_k) \tag{12}$$

is the lth Fourier component of the $v_y(\vartheta_k)$, the y component of electron band velocity, and $\vartheta_k=k_{0y}a$; D is defined with Eq.

(4). Since the electron velocity component $v_y(\vartheta_k)$ depends on details of the SL miniband structure, it is then clear that the band-structure dependence appears explicitly in the SE probability amplitude through the integral I_l of Eqs. (11) and (12). From this general expression of Eq. (11), it is immediately apparent that for electron dynamics in a purely harmonic miniband, that is, one in which only one single Fourier component of velocity, $v_y(\vartheta_k)$, is nonvanishing so that $l=1$, then only the probability amplitude corresponding to the fundamental Bloch frequency contributes to the total emission process.

IV. TOTAL SPONTANEOUS EMISSION RADIATION AND BAND STRUCTURE CHARACTERISTICS

The analysis for total SE radiation characteristics is now developed by considering a general form of the electron energy miniband dispersion relation expressed as

$$\varepsilon_{n_0}(\mathbf{K}) = \varepsilon_{n_0}(0) + \sum_{l=1}^{\infty} \Delta_l \sin^2 \frac{laK_y}{2} + \varepsilon_{\perp}(\mathbf{K}_{\perp}), \tag{13}$$

where $\varepsilon_{n_0}(0)$ is the miniband edge, Δ_l is the width of the lth miniband harmonic of the SL, and $\varepsilon_{\perp}(\mathbf{K}_{\perp})$ is the contribution from the perpendicular components of the band. Such a form of the energy-band dispersion in the SL growth direction generally includes long-range coupling over the neighboring QWs with a relative strength measured by the specific value of the ratio $\Delta_{l+1}/\Delta_l<1$, which is strongly dependent upon the extent of wave-function overlap. In particular, for the well-known case of nearest-neighbor tight-binding (NNTB) approximation, only Δ_1 with $l=1$ with purely harmonic energy dispersion is considered significant, so that next-nearest-neighbor and longer range QW wave-function overlaps are assumed to be negligibly small. The electron group velocity in the general miniband of Eq. (13), for the given K_y in the y direction, is then given by $v_y(K_y)=(1/\hbar)\times[\partial\varepsilon_{n_0}(K_y)/\partial K_y]=\sum_{l=1}^{\infty}v_l\sin(laK_z)$, where $v_l=la\Delta_l/2\hbar$, the maximum velocity associated with the lth miniband of band width, Δ_l. Substituting the expression for $v_y(K_y)$ into Eq. (12), one can find that $I_l=-iv_l/2$. Then the probability amplitude in Eq. (11) reduces to

$$|A_{q_l}(\mathbf{k}_0,\tau_B)|^2 = \pi^2|D|^2\frac{\omega_c v_l^2}{\omega_B^3 l}, \tag{14}$$

where account has been taken for the wave vector $q_l=q_x l\omega_B/\omega_c$. Then, from Eqs. (10) and (14), again using $q_l=q_x l\omega_B/\omega_c$, the total SE probability becomes

$$P_e^s = 2\alpha N\varepsilon^{1/2}\frac{L_x}{L_y}\frac{\omega_c^2}{\omega_B^2}\sum_{l=1}^{l_{max}}\frac{(v_l/c)^2}{l[1-(\omega_c/l\omega_B)^2]^{1/2}}. \tag{15}$$

This is a general expression for total SE probability which contains contributions from higher harmonics of Bloch frequency. Note that in the NNTB approximation of Eq. (13), obtained by letting $\Delta_l=\Delta_1\delta_{1l}$, so that $v_l=v_1\delta_{1l}$, it follows from Eqs. (15) that

$$P_e^s(l=1) = 2\alpha N\frac{L_x}{L_y}\frac{v_1^2}{c^2}\frac{\varepsilon^{1/2}\omega_c^2}{\omega_B^2(1-\omega_c^2/\omega_B^2)^{1/2}}, \tag{16}$$

where $v_1=a\Delta_1/2\hbar$. For a general SL miniband, higher-order harmonic terms beyond the first harmonic term of the Bloch frequency appear in the SE probability amplitude [Eq. (14)], and thus appear in the total SE probability [Eq. (15)] due to a nonzero contribution from higher Fourier components of $v_y(\vartheta_k)$ [Eq. (12)] that are in resonance with corresponding harmonics of the miniband energy spectrum. In Eq. (15), $v_l=la\Delta_l/2\hbar$ indexes the strength of the lth harmonic through the parameter Δ_l, and lth harmonic is indexed through $l\omega_B$. In general, high-order contributions are much weaker than the fundamental one because the coefficients Δ_l in Eq. (13) rapidly decrease with increasing l.[13,14] Note that the SE probability of the higher harmonics in Eq. (15) can be enhanced by tuning the emission frequency $\omega_q=l\omega_B$ to align with the spectral peak of the waveguide TE_{10} mode, so that it is close to the waveguide cutoff frequency ω_c; then, the resonance at the fundamental Bloch frequency will be suppressed because it is less than the waveguide cutoff frequency ($\omega_B<\omega_c$). In particular, for $l=2$, the SE probability for the *second harmonic generation* is expressed as

$$P_e^s(l=2) = \alpha N\frac{L_x}{L_y}\frac{v_2^2}{c^2}\frac{\varepsilon^{1/2}\omega_c^2}{\omega_B^2[1-(\omega_c/2\omega_B)^2]^{1/2}}, \tag{17}$$

where $v_2=a\Delta_2/\hbar$. It is noted therefore that *without scattering* higher-order harmonic generation in the SE spectrum of the accelerated electron from the SL miniband is a signature for band anharmonicity in the SL dispersion.

In noting that the SE probability of Bloch radiation into free space is given by the expression $P_{fs}^s=(2\pi/3)\alpha N(v_1/c)^2$,[1] we can compare both the probabilities for SE at fundamental Bloch frequency into free space and a microcavity analyzing the ratio

$$\eta \equiv \frac{P_e^s}{P_{fs}^s} = \frac{3L_x\omega_c^2\varepsilon^{1/2}}{\pi L_y\omega_B^2[1-(\omega_c^2/\omega_B^2)]^{1/2}}, \tag{18}$$

where use has been made of Eq. (16) for the $P_e^s=P_e^s(l=1)$. The enhancement factor η given in Eq. (18) is a function of the frequency ratio, ω_B/ω_c, and increases monotonically with decreasing ratio; of course, ω_B/ω_c reflects the detuning of the Bloch frequency relative to the TE_{10} mode cutoff frequency. Taking for an estimation $\varepsilon=12.2$ and $L_x/L_y=2$, one can find that for $\omega_B/\omega_c<1.15$ the enhancement factor η increases precipitously with decreasing ratio; at $\omega_B/\omega_c\simeq1.15$, $\eta\simeq10$, indicating that when ω_B is tuned to within $1.15\omega_c$, the SE enhancement will be increased at least by an order of magnitude over unity.

V. SPONTANEOUS EMISSION POWER ESTIMATE

For numerical estimations, we assume a GaAs-based SL structure with the SL lattice parameter $a=100$ Å, vertical dimension 9 μm, and lateral cross section 18×1000 μm^2 embedded into a rectangular waveguide with horizontal and vertical dimensions $L_x/L_y=2$. The electron density in the active region is taken to be 5×10^{16} cm^{-3}. Taking for the SL lowest miniband energy width $\Delta=20$ meV, the maximum group velocity in the miniband is estimated as $v_{max}=1.6\times10^{7}$ cm/s. These parameter magnitudes are close to those of GaAs-based SL structures used to study high-frequency microwave generation.[15–17] Spontaneous emission of a photon with the energy 10 meV corresponds to the Bloch frequency $\nu_B=\omega_B/2\pi=2.5$ THz. The electric field required to achieve such Bloch frequency is $E=\hbar\omega_B/ea=10$ kV/cm, and results in the application of 9 V across the vertical dimension of the SL structure.[18] The spontaneous emission probability of radiation into free space can be estimated taking, for example, $N=100$ as $P_{fs}^{s}=4.3\times10^{-7}$; and the generation energy per electron $\hbar\omega_B P_{fs}^{s}=4.3\times10^{-6}$ meV. Since there is a total of $n=8\times10^{9}$ electrons in the active region of the SL, the generated energy achievable is estimated to be $P_{fs}=n\hbar\omega_B P_{fs}^{s}=34.4$ eV, which corresponds to a power output generated into free space $W_{fs}=(\nu_B/N)P_{fs}\simeq0.14$ μW. Although the power generated into free space is discernibly low for the SE of Bloch oscillation radiation, it follows from Eq. (18) that SE probabilities and rates are substantially modified for the case of radiation into the waveguide mode. We find from Eq. (18) that $\eta=20$, if we take for the detuning parameter $\omega_B/\omega_c=1.05$. Then, using the obtained value for the enhancement factor, we estimate the power output generated into the TE_{10} waveguide mode as $W_{wg}\simeq3$ μW. For purposes of this estimate, the cavity was assumed to be lossless; further studies will consider cavities with finite quality factor.

It is noted that a Bloch oscillation SL does not require controlled inversion population between Wannier-Stark ladder levels to get the desired SE photon frequency; the desired frequency is controlled by the applied field. Whereas in other SL light generating devices, such as quantum cascade lasers, a large inversion population is required to provide stimulated emission with resulting high threshold current densities and high heat dissipation. In this regard, the Bloch oscillator in SE offers a novel option for operating at THz frequencies, provided the power output can be enhanced in the coherent Bloch regime.

VI. SUMMARY

A quantum theory has been presented to describe spontaneous emission of radiation for a Bloch electron accelerating across a single energy band of a SL in a constant electric field while the radiating SL interacts with a resonant microcavity. In the theoretical analysis, use was made of the instantaneous eigenstates of the Bloch Hamiltonian in the external electric field described in terms of the vector potential gauge, and the spontaneous emission rates were calculated using first-order time-dependent perturbation theory in the TE_{10} waveguide mode quantized radiation field. The analysis of spontaneous emission amplitude resulted in transition selection rules for both photon emission frequency and wave number at the Wannier-Stark ladder levels; this result is found naturally through the use of the instantaneous eigenstates of the Bloch Hamiltonian with no *ad hoc* assumptions employed concerning the existence of Wannier-Stark quantized energy levels within the band.

Using the selection rules, the total spontaneous emission probability was calculated for a general SL miniband dispersion, and was shown to depend upon the fundamental and higher-order Bloch frequency harmonics as well as the bandstructure parameters; in the limit of nearest-neighbor tight binding, the results for total spontaneous emission reduced to the expected sole dependence upon the fundamental Bloch harmonic.

Finally, it was shown, within the nearest-neighbor tight-binding model, that the total spontaneous emission probability in the microcavity is substantially enhanced over the comparable free-space value when the Bloch frequency is tuned by means of the external electric field so as to align with the spectral peak of the waveguide mode density of states. In this regard, a theoretical estimate showed that about a one-order-of-magnitude enhancement factor for GaAs-based SLs resulted in a power output of about 3 μW. Here, it is noted that the prime source of the total spontaneous emission enhancement is due to the alignment of the Bloch frequency with the peak of the spectral density of states; in the spirit of the Purcell effect,[2] this will be a general feature of any cavity considered for this analysis.

Future directions of this work include studying the limiting effects of dephasing inhomogeneities such as SL interface scattering and other competitive processes[20–22] so as to more realistically evaluate the optimal magnitudes of power output from the spontaneous emission of SL Bloch oscillation radiation.

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[1] V. N. Sokolov, L. Zhou, G. J. Iafrate, and J. B. Krieger, Phys. Rev. B **73**, 205304 (2006).

[2] E. M. Purcell, Phys. Rev. **69**, 681 (1946).

[3] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).

[4] In *Spontaneous Emission and Laser Oscillation in Microcavities*, edited by H. Yokoyama and K. Ujihara (CRC, Boca Raton, 1995).

[5] C. Janke, P. Haring Bolivar, A. Bartels, H. Kurz, and H. Künzel, Phys. Rev. B **67**, 155206 (2003); M. Zedler, C. Janke, P. H. Bolivar, H. Kurz, and H. Künzel, Appl. Phys. Lett. **83**, 4196 (2003).

[6] G. Gallot, S. P. Jamison, R. W. McGowan, and D. Grischkowsky, J. Opt. Soc. Am. B **17**, 851 (2000).

[7] D. Marcuse, *Principles of Quantum Electronics* (Academic, New York, 1980), p. 426.

[8] N. Marcuvitz, *Waveguide Handbook* (Peregrinus, London, 1993).

[9] J. B. Krieger and G. J. Iafrate, Phys. Rev. B **33**, 5494 (1986); G. J. Iafrate and J. B. Krieger, *ibid.* **40**, 6144 (1989).

[10] J. Singh, *Quantum Mechanics: Fundamentals and Applications to Technology* (Wiley-Interscience, New York, 1997), p. 337.

[11] Note that $\mathbf{q}$ is not collinear with the direction of the applied field $\mathbf{E}$, then the integral in Eq. (7) does not vanish because of the periodicity of $\varepsilon_{n_0}(\mathbf{k}_0)$, and contributes to the selection rule. For $\omega_q=\omega_c[1+(q_z/q_x)^2]^{1/2}$, this corresponds to neglecting a small term $\sim v_\perp/c$ as compared to unity, where $v_\perp$ is the electron band velocity perpendicular to the y axis.

[12] In the photon long-wavelength limit, the probability amplitude in Eq. (4) allows one to factor out the arbitrary initial value of the electron wave vector $\mathbf{K}_0$ resulting from t_0 as a phase factor, so that $A_{q_l}(\mathbf{k}_0,\tau_B)=-D(q_x/q_l)^{1/2}(2\pi/\omega_B)\exp(ilaK_{0y})I_l$, with I_l defined in Eq. (12). Therefore the absolute value of A_{q_l} which gives $|A_{q_l}(\mathbf{k}_0,\tau_B)|^2$ in Eq. (11) is independent of t_0 or $\mathbf{K}_0$, a consequence of periodicity property of the band.

[13] As noted in Refs. 14, higher Bloch frequency harmonic terms that are much weaker than the fundamental term may appear in a pure harmonic energy miniband due to the interaction of Bloch electrons with plasmons or with cyclotron oscilations in an external magnetic field; these terms arise as higher harmonic terms in the semiclassical description of oscillating current. Such effects as well as other effects imposed on dephasing Bloch oscillations due to electron scattering are not treated in this paper.

[14] F. Löser, Yu. A. Kosevich, K. Köhler, and K. Leo, Phys. Rev. B **61**, R13373 (2000); A. B. Hummel, T. Bauer, E. Mohler, and H. G. Roskos, J. Phys.: Condens. Matter **18**, 2487 (2006).

[15] L. Esaki and R. Tsu, IBM J. Res. Dev. **14**, 61 (1970).

[16] E. Schomburg, T. Blomeier, K. Hofbeck, J. Grenzer, S. Brandl, I. Lingott, A. A. Ignatov, K. F. Renk, D. G. Pavel'ev, Yu. Koschurinov, B. Ya. Melzer, V. M. Ustinov, S. V. Ivanov, A. Zhukov, and P. S. Kop'ev, Phys. Rev. B **58**, 4035 (1998).

[17] A. M. Bouchard and M. Luban, Phys. Rev. B **47**, R6815 (1993).

[18] To prevent charge domain formation that may be possible through negative differential conductance a superstructure of SLs of suitable lengths can be designed (Ref. 19).

[19] P. G. Savvidis, B. Kolasa, G. Lee, and S. J. Allen, Phys. Rev. Lett. **92**, 196802 (2004).

[20] N. Sekine, Y. Shimada, and K. Hirakawa, Appl. Phys. Lett. **83**, 4794 (2003).

[21] J. P. Reynolds and M. Luban, Phys. Rev. B **54**, R14301 (1996).

[22] G. von Plessen, T. Meier, J. Feldmann, E. O. Göbel, P. Thomas, K. W. Goossen, J. M. Kuo, and R. F. Kopf, Phys. Rev. B **49**, R14058 (1994).